Explicitly Valuing Mathematical Creativity in Proof-Based Courses

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• Wolfram Alpha
• Web sites dedicated to textbook solutions
• Automatic theorem-provers
• Workforce change in the United States
  • More than 1,500 Chief Executive Officers from 60 countries and 33 industries worldwide, believe that -- more than rigor, management discipline, integrity or even vision -- successfully navigating an increasing complex world will require creativity. (IBM 2010 Global CEO Study)
MAA CUPM 2015 Guidelines

- A successful major offers a program of courses to gradually and intentionally leads students from basic to advanced levels of critical and analytical thinking, while encouraging creativity and excitement about mathematics.

- Major programs should include activities designed to promote students’ progress in learning to approach mathematical problems with curiosity and creativity and persist in the face of difficulties.
“It is in the best interest of the field of mathematics education that we identify and nurture creative talent in the mathematics classroom” (Sriraman, 2004, p. 32).

“For it is through mathematical creativity that we see the essence of what it means to ’do’ and learn mathematics.” (Liljedahl, 2009, p. 239).
What is mathematical creativity?
In particular, in proving?

Over 100 definitions of mathematical creativity!
(Mann, 2006)
Defining Creativity

• Psycho-Analytic: Many mathematicians describe an enlightenment that is somewhat unexpected (Hadamard, 1945; Poincare, 1958).

• Product: Some focus on emphasizing whether the end-product is original and useful (Runco & Jaeger, 2012), perhaps to the mathematics field (Csikszentmihalyi, 1988).

• Process: describe it as a process that involves different modes of thinking, some of an unusual nature (Balka, 1974).
• Absolute creativity versus Relative creativity
  • Historical inventions or discoveries at a global level
  • The discoveries by a specific person within a specific reference group, to human imagination that creates something new (Vygotsky, 1982, 1984)
• Torrance (1966, 1978) created testing for creativity and giftedness in K-12 education
• Silver (1997) expanded three aspects of K-12 mathematical creativity
  • Flexibility - An ability to look at a problem from new perspective
  • Originality - Using an unexpected or unusual approach
  • Fluency - Applying ideas, tools of one area in a different area
• Elaboration – Expanding on each approach (Torrance, 1978)
• Iconoclasm – Affect, Self-efficacy of each approach (Chamberlain & Mann, 2015)
• Creativity in K-12 classrooms is different than the kind employed by mathematicians (Sriraman, 2005)
• Proof-writing aligns more with mathematicians’ creativity
Overarching Research Question

How can we explicitly value and foster potential for undergraduate students’ creativity in mathematics?
• What properties/actions can undergraduate students learn/enact to generate potential for being creative in proving?
• How can we implement said properties/actions in the classroom?
Creativity-in-Progress Rubric (CPR) on Proving

- Creativity rubric from AAC&U (Rhodes, 2010)
- Leikin’s (2009) Problem-Solving Rubric
- Interview with mathematicians (Tang, et al., 2015)
- Constant alpha-testing on students’ LiveScribe work
- Feedback from past presentations
- Coding student process with mathematicians
CPR on Proving (cont.)

- **Categories**
  1. Making Connections
  2. Taking Risks

- **Levels (Continuum)**
  - Beginning
  - Developing
  - Advancing
<table>
<thead>
<tr>
<th>MAKING CONNECTIONS:</th>
<th>Beginning</th>
<th>Developing</th>
<th>Advancing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Definitions/Theorems</td>
<td>Recognizes some relevant definitions/theorems from the course or textbook with no attempts to connect them in their proving</td>
<td>Recognizes some relevant definitions/theorems from the course and attempts to connect them in their proving</td>
<td>Implements definitions/theorems from the course and/or other resources outside the course in their proving</td>
</tr>
<tr>
<td>Between Representations</td>
<td>Provides a representation with no attempts to connect it to another representation</td>
<td>Recognizes connections between representations</td>
<td>Uses connections between different representations</td>
</tr>
<tr>
<td>Between Examples</td>
<td>Generates one or two specific examples with no attempt to connect them</td>
<td>Recognizes a connection between the generated examples</td>
<td>Uses the key idea synthesized from generating examples</td>
</tr>
</tbody>
</table>
## CPR on Proving

<table>
<thead>
<tr>
<th><strong>TAKING RISKS:</strong></th>
<th><strong>Beginning</strong></th>
<th><strong>Developing</strong></th>
<th><strong>Advancing</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tools and Tricks²</td>
<td>Uses a tool or trick that is algorithmic or conventional for the course or the student</td>
<td>Uses a tool or trick that is model-based or partly unconventional³ for the course or the student</td>
<td>Creates a tool or trick that is unconventional for the course or the student</td>
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<tr>
<td>Flexibility⁴</td>
<td>Begins a proof attempt (or more than one proof attempt), but uses only one approach</td>
<td>Acknowledges and/or uses more than one proving approach, but only draws on one proof technique</td>
<td>Uses more than one proof technique</td>
</tr>
<tr>
<td>Posing Questions</td>
<td>Recognizes there should be a question asked, but does not pose a question⁵</td>
<td>Poses questions clarifying a statement of a definition or theorem</td>
<td>Poses questions about reasoning within a proof</td>
</tr>
<tr>
<td>Evaluation of Proof Attempt</td>
<td>Examines surface-level⁶ features of a proof attempt</td>
<td>Examines an entire proof attempt for logical or structural flow</td>
<td>Examines and <em>revises</em> an entire proof attempt for logical or structural flow</td>
</tr>
</tbody>
</table>
What the CPR on Proving is and is not

- It is NOT assessing “correctness” or “validity” of the final proof.
- It is examining the full process of proof production.
- It is NOT a rubric to label student’s creativity!
- It makes explicit some aspects that may promote mathematical creativity.
• Non-judgmental environment
• Authority in creativity
• “To promote creativity and mathematical thinking, teachers should encourage good ideas even (and, in fact, especially) when a student suggests an unexpected answer or when the answers are inaccurate.” (Hershkovitz, Peled, & Littler, 2009, p. 265)
Problem Posing/Conjecturing

Solutions as problems: “Students are rarely asked to view a solution to a problem as a starting point in problem solving” (Knuth, 2002, p. 129)
The organizational climates that stimulate creativity (Amabile, 1988; Isaksen, 1995):
- feel challenged by their goals, operations and tasks
- feel able to take initiatives and to find relevant information
- feel able to interact with others
- feel that new ideas are met with support and encouragement
- feel able to put forward new ideas and views
- experience much debate within a prestige-free and open environment
- feel uncertainty is tolerated and thus risk-taking is encouraged.
Two major actions that create potential for mathematical creativity:
- Making Connections
- Taking Risks

Our conjecture is that, if both are fostered in any environment, creative (and eventually valid) products will be produced.
Future Research

- Implementation of CPR on Proving in the classroom
- Creativity x Neuroscience
  - EEG “reliving” with the proving process
- CPR on Problem Solving
- Creativity in a social setting
- CPR on Linear Algebra, Calculus, and Pre-Calc?!?
It must not be forgotten that the basic law of children’s creativity is that its value lies not in its results, not in the product of creation, but in the process itself. It is not important what children create, but that they do create, that they exercise and implement their creative imagination. (Vygotsky, 2004, p. 72)

Thank you!

Questions?
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