Tasks to Foster Mathematical Creativity in Calculus I

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Fostering students' mathematical creativity necessitates certain instructional actions - one of which is designing and implementing tasks that foster creativity. Drawing on the literature on mathematical creativity, we describe existing research-based features of tasks for eliciting student creativity, or creativity-based tasks, and provide suggestions for implementation of such tasks. Based on these features, we analyzed two instructors' first experiences designing and implementing creativity-based tasks in Calculus I. Both instructors' frequent use of the multiple-solutions feature suggests that this feature could be an entry-point for designing and implementing creativity-based tasks for other instructors seeking to foster creativity.

Keywords: Calculus, creativity-based tasks, mathematical creativity, task design

The importance of mathematical creativity in mathematics and mathematics courses is documented in numerous research studies, policy and curriculum-standard documents (e.g., Borwein, Liljedahl, & Zhai, 2014; CUPM, 2015; Levenson, 2013; Moore-Russo & Demler, 2018; NSB, 2010; Silver, 1997; Sriraman, 2009; Tang et al., 2017; Zazkis, & Holton, 2009). Askew (2013) points out that "[c]alls for creativity within mathematics and science teaching and learning are not new, but having them enshrined in mandated curricula is relatively recent" (p. 169). For example, in its latest guidelines for majors in mathematical sciences, the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (CUPM) states that "these major programs should include activities designed to promote students' progress in learning to approach mathematical problems with curiosity and *creativity* [emphasis added] and persist in the face of difficulties" (Schumacher & Siegel, 2015, p. 10). In this paper, we focus on mathematical creativity in Calculus I, a course that is commonly offered for majors in these mathematical sciences programs.

Students' experiences in Calculus I play a critical role in their persistence in science, technology, engineering, and mathematics (STEM) programs (Rasmussen et al., 2019). The lack of or limited exposure to class materials (e.g., tasks, homework problems, exam questions) that promote conceptual discussions is one of the reasons reported by Calculus I students for switching out of a STEM major (Johnson, Ellis, & Rasmussen, 2014). In fact, a textbook analysis (Lithner, 2004) concluding that 70% of Calculus exercises at the end of the section were about mimicking previously done examples of the same section is one indication to students' limited exposure to conceptual ideas. Though there are Calculus reform projects that address this particular issue by advocating for new curriculum materials for more conceptual discussions (see Bressoud et al., n.d.), we believe it is still a priority to explicitly value and foster students' mathematical creativity in Calculus I classes. In particular, we argue for the creation and implementation of tasks that are designed not only for conceptual understanding but also for enhancing students' mathematical creativity. In this paper, we discuss research-based features of tasks that promote mathematical creativity and their potential implementation. Additionally, we share analysis of two instructors' uses of these features in the design and implementation of the tasks in their Calculus I classes.

Theoretical Perspective and Background Literature

In our work, we define mathematical creativity as a process of offering new solutions or insights that are unexpected for the student with respect to their mathematics background or the problems they have seen before (Liljedahl & Sriraman, 2006; Savic et al., 2017). This process-oriented definition (Pelczer & Rodriguez, 2011), in contrast to examining final products (Runco & Jaeger, 2012) of those processes, provides a dynamic view of creativity rather than a static one. We focus on valued mathematical actions (Cuoco, Goldenberg, & Mark, 1996; Schoenfeld, 1992) such as taking risks and making connections that can lead to creativity in mathematics (Karakok et al., 2015; Leikin, 2009). Our definition also encompasses creativity relative to the student versus creativity relative to the field of mathematics (Beghetto & Kaufman, 2013; Leikin, 2009). Finally, this particular definition identifies creativity specific to the domain of mathematics rather than domain-general creativity (Baer, 1998).

Fostering students' mathematical creativity, as we define it, necessitates certain instructional actions that are encapsulated by Sriraman's (2005) five theoretical principles. The *Gestalt principle* discusses the importance of giving time to allow incubation to occur (Hadamard, 1945). The *Aesthetic principle* highlights explicitly valuing the beauty and uniqueness of solutions or methods. The *Free-market* and *Scholarly principles* emphasize creating a safe environment where students can present and defend their work, and allowing students to build off one another's work, respectively. The fifth principle, *Uncertainty*, focuses on tolerating ambiguity and knowing that it is acceptable to not know a solution. Levenson (2011) provides empirical support for these principles and adds "choosing appropriate tasks" (Levenson, 2013, p. 273) as one of the roles of teachers for the promotion of creativity.

We conjecture that intentionally implemented tasks designed to align with Sriraman's principles have the potential to enhance mathematical creativity. For example, a task that is aligned with the Gestalt principle needs to be challenging enough for students that they would need time to incubate. An intentional implementation of such a task that demonstrates the Scholarly principle would include an instructor giving students the opportunities to discuss their approaches and build off of one another's work; all the while, students' mathematical creativity is at the forefront of such discussions.

Task design

We adopt Henningsen and Stein's (1997) definition of a mathematical task as ''a classroom activity, the purpose of which is to focus students' attention on a particular mathematical concept, idea, or skill'' (p. 528). For a task to promote creativity, it needs to have additional features that provide students opportunities to push their mathematical processes toward new solutions or insights that are unexpected for them. For practical reasons, we use the term *creativity-based tasks* to describe such tasks.

We situate our discussion of creativity-based tasks and task design within two perspectives of creativity: *Developmental* and *Problem Solving and Expertise-Based* perspectives (Kozbelt, Beghetto, & Runco, 2010). The *Developmental* perspective posits that creativity develops over time (i.e., process-orientation) in an environment where students are provided authentic tasks and opportunities to interact with others. The *Problem Solving and Expertise-Based* perspective emphasizes problem-solving processes, heuristics, and tasks, underscoring the use of tasks to challenge students' thinking processes and provide opportunities to solve problems in various ways. A key component of our work is developing tasks that would allow such processes. We base these tasks on Skemp's (1976) relational understanding framework and Lithner's (2008) creative mathematically-founded reasoning. As relational understanding relates to students' development of conceptual structures, a creativity-based task promotes both making connections between concepts and taking risks by students to become independent thinkers. Additionally, creative mathematically-founded reasoning involves novel (with respect to students) mathematical arguments. It is noted by Boesen, Lithner, and Plam (2010) that students use creative mathematically-founded reasoning to solve unfamiliar, nonroutine tasks. In this sense, creativity-based tasks can be viewed as unfamiliar and non-routine.

Features of Creativity-Based Tasks

"Recall and apply" tasks are important in developing procedural fluency in Calculus I, but to foster mathematical creativity, instructors need to design tasks that require "evaluating mathematical statements; example generation (constructing an instance); analyzing reasoning; conjecturing; generalizing; visualization; using definitions" (Breen & O'Shea, 2011, p.87). Tasks with these features can promote conceptual discussions and making connections between seemingly different ideas and concepts.

Silver (1997) discusses the importance of the interplay between problem posing and problem solving to creativity and states "[i]t is in this interplay of formulating, attempting to solve, reformulating, and eventually solving a problem that one sees creative activity" (p. 76). We believe it is important for tasks to engage students in problem posing and problem solving not only in order to promote creativity but also to enable "teachers and students to become subjects of the educational process by overcoming authoritarianism and an alienating intellectualism" (Freire, 1999, p. 8). The need for posing problems can be facilitated by assigning tasks that are ill-defined, ambiguous, or open-ended. Kwon et al. (2006) define an incomplete or an open-ended problem as "a problem which does not define clearly what the question asks for, therefore allowing many possible solutions" (p. 52). Thus, another feature of a creativity-based task is providing opportunities for students to pose problems and questions, then to seek answers to these problems and questions (Haylock, 1997; Silver 1997). Experts describe the ability to identify key research questions as part of their creative work (e.g., Hadamard, 1945; Mansfield & Busse, 1981).

Relating to many possible solutions, Leikin (2013) defines a multiple-solution task as one that "explicitly requires students to solve a mathematical problem in different ways" (p. 388) where different solutions are determined by: "(a) different representations of some mathematical concepts involved in the task, (b) different properties (definitions or theorems) of mathematical objects within a particular field, or (c) different properties of a mathematical object in different fields" (Leikin, 2013, p. 388). Thus, multiple-solution tasks can also promote utilizing other representations (verbal, symbolic, gestures) as well as connecting certain aspects of different representations in a way that fosters deeper mathematical thinking. Multiple-solution tasks not only value students' individual approaches, but they also allow for originality and novelty in using certain standard or less standard tricks.

Examples

One of the tasks that we designed for Calculus I, the *Circle Task* poses the questions "Is there anything in real-life that is a perfect circle? How do you know if you have a perfect circle?" This task involves the mathematical concepts of infinitesimals, (possibly) limits, integrals, and arc length. As a creativity-based task, its open-ended nature provides opportunities for students to take risks in exploring novel ideas and to make conceptual connections to fundamental aspects of Calculus.

As another example, we modified a typical "find the limit" question to "Consider the limit $\lim_{x \to 1} \frac{\sqrt{x}-1}{x-1}$. Evaluate the limit in as many ways as possible." We designed this *Limit Task* as a creativity-based task that involves the mathematical concepts of limits and derivatives. By asking students to evaluate the limit in more than one way, students are pushed to think of a solution beyond one that they are mathematically inclined to provide. As the question did not specify how students are to approach the problem (algebraically, graphically, using a table, etc), the task carries the open-ended feature. The task fosters making connections between various concepts as students could view this limit as a slope of the tangent line, which could be computed by the derivative at 1.

Task Implementation

Stein, Grover and Henningsen (1996) noticed that tasks that were designed to be cognitively demanding (e.g., involving conjecturing, justifying, generalizing) became less demanding because they "became routinized, either through students' pressing the teacher to reduce task ambiguity and complexity by specifying explicit procedures or steps to perform or by teachers' taking over the challenging aspects of the task" (p. 479). That is to say, the implementation of a task plays a crucial role in fostering creativity.

To mitigate the possibility of a reduction in cognitive demand, we suggest using Sriraman's (2005) five principles as guidelines for implementing creativity-based tasks. To bridge theory and practice, the authors (Cilli-Turner et al., 2019) investigated one teacher's actions with the five principles and suggested numerous teacher actions stemming from these principles to potentially foster students' mathematical creativity. Building on the results from (Cilli-Turner et al., 2019), we suggest that when implementing an open-ended task such as the Circle Task, instructors could assuage students' discomfort in task ambiguity (Uncertainty) by assuring students that multiple solutions exist. The instructors can also provide additional time for students to incubate (Gestalt) on what it means to have a perfect circle and a real-life circle. Following this incubation, instructors can give students opportunities to present their findings (Free Market and Scholarly). Similarly, the Limit Task affords different possible implementations because it has a "call-back" feature where instructors can use it progressively at various points throughout the semester as new material is covered, deepening students' incubation period (Gestalt). Aside from (re)using this task while covering graphs, limits, and derivatives, the task can be revisited after covering L'Hospital's Rule by asking students to find the limit in four different ways and giving students opportunities to share (Free Market and Scholarly) these different ways. Instructors can facilitate the Aesthetic principle by valuing the novelty of using the trick of factoring linear terms in this limit.

Research Methods

As part of a larger research study that explores mathematical creativity in Calculus I classes, our research team designed the two creativity-based tasks: the Circle Task and Limit Task. These two tasks and their features were shared with two instructors, Jo Parker and Juniper Travers (pseudonymous), at a South-Midwest regional university who participated in the larger study. Prior to the start of the 'research' semester, we had 2 two-hour online professional development (PD) sessions with a two-day break between the sessions. At these sessions, we introduced some goals of the research project, features of creativity-based tasks, and the Limit Task. The sessions also included discussions on various ways to explicitly value and foster mathematical creativity and implement creativity-based tasks. The two instructors designed a task or modified an existing one to include some creativity-fostering features. To provide extended support, the online PD continued on a weekly basis throughout the semester for an hour to support participants' instructional practices that explicitly foster mathematical creativity and task design. We facilitated open-ended discussions concerning how to assess student work on creativity-based tasks. Both instructors were asked to implement the Limit and Circle tasks in their Calculus I classes and also to develop and implement at least four other creativity-based tasks.

In this paper, we share results from our preliminary analysis of collected data to address the research question: What features do instructors use in their creativity-based task design and implementation processes? We collected instructors' creativity-based tasks, their journal entries for the PD in which they reflected on their design and implementation processes, their classroom video-recording of the days on which these tasks were used in class, and their Calculus I material from previous teaching experiences. At the end of the semester, we conducted semi-structured interviews that included questions regarding their tasks and task design processes. Instructors' tasks, journal entries, and interview transcripts were analyzed with a deductive approach using task features as codes (Patton, 2002).

These codes or features (in *italic*) were: *open-ended*; allows *multiple solutions, multiple representations, different approaches leading to one answer, posing problems and questions;* promotes *making connections between different concepts, evaluation/justification, generalization making, incubation;* allows for *originality/novelty, conjecturing, use of a trick or a less standard algorithm,* and *uncertainty*.

Analysis

We analyzed the tasks that instructors used as creativity-based tasks in their courses. Some of these tasks were only one problem in a longer activity sheet or an assignment set. The implementation of these tasks varied from task to task and instructor to instructor. We observed that both instructors most frequently used the "multiple solutions" feature in their tasks. These multiple-solution tasks also afforded multiple representations (algebraic, symbolic, graphical). For example, Juniper developed this extrema task:

(a) Sketch or write the equation of a function for which the 2nd derivative test is inconclusive at $x = 1$. Provide justification as to how you know the 2nd derivative test fails.

(b) What would be a next step for finding extrema if the 2nd derivative fails?

We coded this task as a *multiple-solution* task that could afford *multiple representations* (algebraic and/or graphical). This task also included features such as requiring students' *originality* in creating such a function, promoting *uncertainty* as students might need to try few possibilities before they find a function that works. We coded the task as one that fosters students' *evaluation* skills as it required them to justify if the test fails. We coded Part (b) of the task as *an open-ended* question where the instructor did not provide specific directions to students.

Similarly, Jo's task on integration "Find a non-trigonometric function f and domain $[a, b]$ such that $\int_a^b f(x)dx = 0$" was coded as a task that has *multiple solutions* and *multiple representations* (algebraic and/or graphical). It was also coded as a task requiring students' *originality* in creating such a function and promoting *uncertainty* as students might need to try a few possibilities before they find a function that works. As a contrast to the extrema task, we did not code this task as an open-ended one because the question instructed students to find such a function, and hence students knew, before tackling the task, that there will be at least one. If the task was phrased “Is there a non-trigonometric function…? If so, find at least one (or two)” then it would have carried the open-ended feature.

In many of Jo’s creativity-based tasks, she asked for creation of a function that satisfies (or does not satisfy) certain criteria. With this structure, the most common feature Jo utilized was *multiple solutions* affording *multiple representations*. The *open-ended* feature was utilized on fewer occasions. The feature of *different approaches leading to one answer* was also utilized but minimally. Most of Jo’s tasks fostered *making connections between different ideas and concepts* and promoted Sriraman’s *uncertainty* principle.

One of the most noticeable changes between materials from her previous Calculus teaching to the ‘research’ semester was the adaptation of creativity-based tasks on her final exam review sheet. On the final review sheet in the previous semester, we coded 15 questions as “*routine*” exercises in Calculus I as they resembled the typical exercises. On the other hand, the final review sheet for the research semester had five “non-routine” creativity-based tasks with the following combination of features: *multiple solutions* and *representations*, *open-ended*, and *different approaches*. The tasks also fostered *making connections between different concepts* and aligned with the *uncertainty* principle. She also added a creativity-based task on her final exam.

Juniper’s creativity-based tasks were developed for each of the following content areas: limits, continuity, relative extrema, absolute extrema, and integration. Many of these tasks asked to create a function that satisfies (or does not satisfy) certain criteria, which were similar to tasks developed by Jo. With this structure, we noticed that Juniper utilized *multiple-solution* tasks affording *multiple representations* most frequently. The *open-ended* feature was also used frequently. Juniper included tasks that required students to *pose problems and questions*. There were questions on tasks that explicitly asked students to *evaluate* or justify their answers or approaches. It was noticeable that her final exam did not include any of her creativity-based tasks and it was very similar to her final exams from previous semesters. However, she did use a creativity-based task on Exam 2.

Participants’ journal entries were also coded for features of tasks. For example, Jo Parker implemented the Circle task as a writing assignment. In her teaching reflection journal, she stated that she wanted to “provide students with the opportunity to practice written communication skills while also gaining experience in answering a more open-ended mathematical question. I told the students that there was not a single correct answer to the prompt…” In this reflection, it seems that Jo Parker embraces the *open-ended* feature of this task to facilitate an additional mathematical process, written mathematical communication skill. Juniper’s reflections hinted that she wanted to emphasize *incubation* in her implementation of tasks. She assigned students to work on some of the tasks outside of class and asked them to bring their work to the next class for discussion.

We also triangulated our coding of participants’ tasks and journal entries with their interview transcripts. When asked “was there any particular feature that was most important to you when you were designing or thinking about these tasks,” Jo referred to the features of *multiple solutions* and *posing questions*. In her implementation of her task in which she asked students to explore the conditions for which Rolle’s theorem holds, she was happily surprised that even after students submitted their answers, they were posing questions and discussing ideas with each other in class. Referring to the Limit task, she said, “I love the fact that you can approach it in many different ways…I think the traditional way that students think of it is multiply by the conjugate. But, I mean they can factor in it. I mean it's just a little trick…I think it's cute. It's a little bit outside the box but it's still within their realm of knowledge."

On the other hand, Juniper referred to the extrema task as a memorable task, stating that "[it] sticks out in my mind because that's where I actually saw [students] construct another concept that we haven't gotten to yet. So that it's like a huge leap where creativity had taken them further in mathematics content." The most common task feature she stated was justification because, "[it] is always important in my mind. They can come up with an answer, but they have to be able to explain… I think underlying all of it was 'do these questions really highlight the underlying mathematical concepts?" Although this was coded for *evaluation/justification* elsewhere in her tasks, the *multiple solutions* feature was coded most often in Juniper's tasks. It could be that in her implementation of these tasks, she provided additional questions during discussions that facilitated this feature of evaluating and justifying.

Discussion

Raising instructors' awareness of mathematical creativity and advocating for change in our pedagogies to promote creativity are the driving purposes of this project. This change can be initiated with small adjustments in task design. The results of our initial analysis of data demonstrate these adjustments made by two instructors. As noted in the analysis, the multiple solutions feature was most commonly used in tasks designed by the instructors. This result mirrors Levenson's (2013) findings that many of her participants advocated for the use of multi-solution tasks. Levenson further noted that the collection of tasks that were chosen by five instructors who thought these tasks fostered mathematical creativity had fewer open-ended tasks and no problem-posing tasks. Our results indicate a similarity, which we believe is due to the fact that this project was the first time for our participants to set out to explicitly value creativity in their classroom; hence there seems to be a limited attempt to having open-ended and problem-posing tasks. Though Levenson (2013) highlights the importance of open-ended tasks and problem-posing tasks claiming that they "may afford additional opportunities for developing students' mathematical creativity and it is important to raise teachers' awareness to the variety of tasks which may serve this purpose" (p. 288).

An intentional selection or creation of tasks that have multiple solutions or that can be approached in different ways seem to be a feasible entry point into fostering mathematical creativity. Allowing students to pose problems, generate examples, make conjectures, and so forth, are other ways to direct students' thought processes towards creative mathematically-founded reasoning (Lithner, 2008). According to Beghetto (2017), instructors can alter their tasks slightly, such as asking students to come up with their own approaches, or substantially by asking students pose their own problems and ways of solving those problems.

According to Sriraman, teaching practices that are aligned with the five principles have the potential of fostering students' creativity. In our analysis of creativity-based tasks, we noticed that the Gestalt and Uncertainty principles can be viewed as features of such tasks. However, we claim that the Free Market, Scholarly, and Aesthetic principles are important aspects of the implementation of such tasks that can further promote students' creativity. In fact, these aspects of implementations differentiate creativity-based tasks from non-routine tasks. We plan to investigate this claim in future research to study potential implementations that complement creativity-based tasks in fostering students' creativity.

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