An Investigation into Sociomathematical Norms of Proof Schemes

*Brian Katz, Augustana College
Rebecca Post, Augustana College
Milos Savic, University of Oklahoma
John Paul Cook, University of Sciences and Arts of Oklahoma

Proof Schemes

- Harel and Sowder (1998)
  - "consists of what constitutes ascertaining and persuading for that person" (p. 244)
  - Three broad types, sorted by sources of evidence
    - External – relies on authority
    - Empirical – appeal to "physical facts or sensory experiences" (p.252)
    - Analytical – logical deductions

Axiomatic Proof Schemes

- “A mathematical justification must have started originally from undefined terms and axioms” (p. 273)
  - Distinction between undefined terms and defined terms
  - Distinction between statements accepted without proof and proofs deduced from other statements
- Call to Action: Educational reform for students to build axiomatic proof schemes

Sociomathematical Norms

- Social norms: Cobb and Yackel (1996); Yackel and Cobb (1996)
  - The "rules and regularities of classroom behavior" (Fukawa-Connelly, 2012, p. 402)
- Sociomathematical Norms
  - Social norms that are "specific to the discipline of mathematics" (Fukawa-Connelly, 2012, p. 403)

Sociomathematical Norms

- Yackel, Rasmussen, and King (2000)
- Martin, Soucy McCrone, Wallace Bower, and Dindyal (2005)
- Fukawa-Connelly (2012)
  - Convincing oneself
  - Justifying new inferences based on previous ones
  - Using only peer-validated knowledge

Research Questions

- What norms are enacted in an undergraduate advanced mathematics classroom that encourage and facilitate classroom discussions about the certainty of mathematical knowledge?
- How do the norms and beliefs support students to engage in appropriate activities for proof-writing or making sense of presented proofs?

Methods - Data

- Upper-level Modern Geometry course at a small liberal arts college
- 75 minutes per day, three days per week, 10 weeks
- 11 mathematics majors, 8 specializing in math ed.
- Class time involved small group proving and large class discussion and evaluation
- Regular written reflections from all students
- Interviews with all students at the end of the course

Methods - Analysis

- Large group discussion transcribed fully
- Borrowed from grounded theory approach (Strauss & Corbin, 1990). Transcriptions were initially coded individually then codes were agreed upon by the group for inter-rater reliability and tested against the transcripts for legitimacy.
- Three different modes of norm instantiation
  - Teacher explicitly *establishing* norm
  - Teacher *prompting* the norm by questioning, students *negotiating* their understanding of the norm
  - Student *enacting* norm
- Small group discussion, interviews, reflections transcribed and coded when episodes related to emergent themes were observed

Results

- One social norm observed: *meta-argumentation*
  - Members of the community are responsible for evaluating approaches to justifying claims and using that evaluation to guide inquiry, proof, and communication.
- Four observed, associated sociomathematical norms
  - N_1 – Objectifying current and potential knowledge
  - N_2 – Analyzing methods for spanning this gap
  - N_3 – Contrasting various disciplinary ways of knowing
  - N_4 – Choosing based on aesthetic differences

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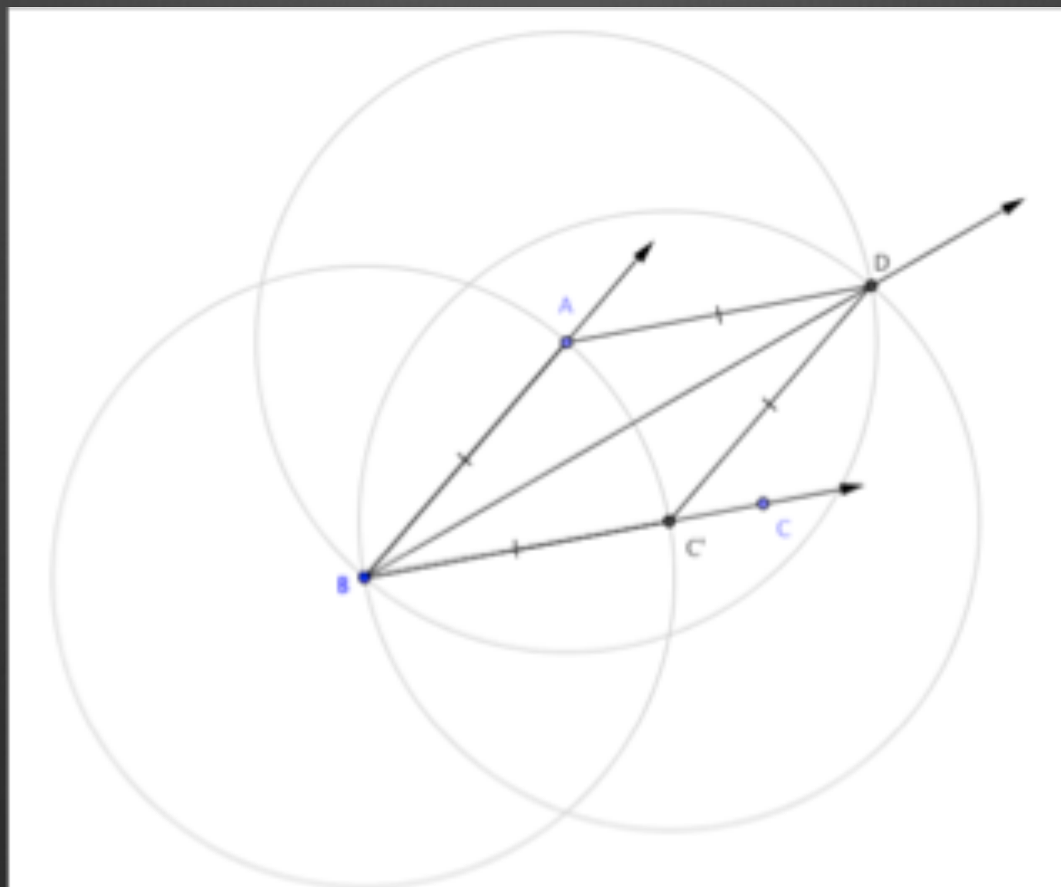
Episode 1: Bisecting an Angle



I: OK. Yep, they're congruent, but why? What results do we have whose conclusion is "therefore these angles are congruent”?

BP: Congruent triangles.

I: Which we call what? [DV]?

DV: Congruent parts of congruent figures are congruent.

I: That's the ONLY result we have so far that says "therefore two angles are congruent", right? So that HAS to be your second to last step. So? Do you know any angles are congruent already?

All: (head shaking)

I: You don't know anything about angles here, so what is our only other... and line segments, we're not going to get there just with line segments, right? We have three assumptions. We have Axioms 1, 2, and 3. Axiom 1 says segments are congruent if their lengths are, but that's never going to get us to figures being congruent, right? We have side-side-side and side-angle-side. What do you think, [GV]?

GV: Side-side-side.

I: Can you put those two angles inside triangles that are congruent by side-side-side? Can you see those two angles as inside two triangles that are congruent by side-side-side?

EJ: Kind of.

KG: Yeah. Because this line (BD) is shared by both the triangles so it's the same.

Episode 2: Constructing a square

- Task "Problem 54. *Construct a square.*"
  - Students had attempted this and other tasks in preparation for class.
  - Students spent approximately 50 minutes working in small groups trying to construct a square with a complete justification of their construction.
  - Instructor moved between groups, asking students to be explicit about their construction and justification.

Episode 2a: The square

(Instructor draws a curved line.)

Student 1: How does a line curve?

Student 2: Don't we assume that all lines are straight?

I: What does straight mean?

Student 2: Seriously? (groans)

I: So lines are undefined. We have never defined a line.

Student 1: So how do we know that they really don't intersect?

I: They might. But we don't know it.

Student 1: OK, so we don't know the relationship between parallel and perpendicular.

Student 3: Wait…if we're just talking about this now, then what have we been doing for like…(laughing)

Episode 2b: The square

I: Not only is this not working, there are no squares here. So what does this mean about what must happen next? This is a more philosophical question, and less mathematical.

Student: We need like more stuff, more theorems to cover for it, kind of?

I: Not more theorems but…

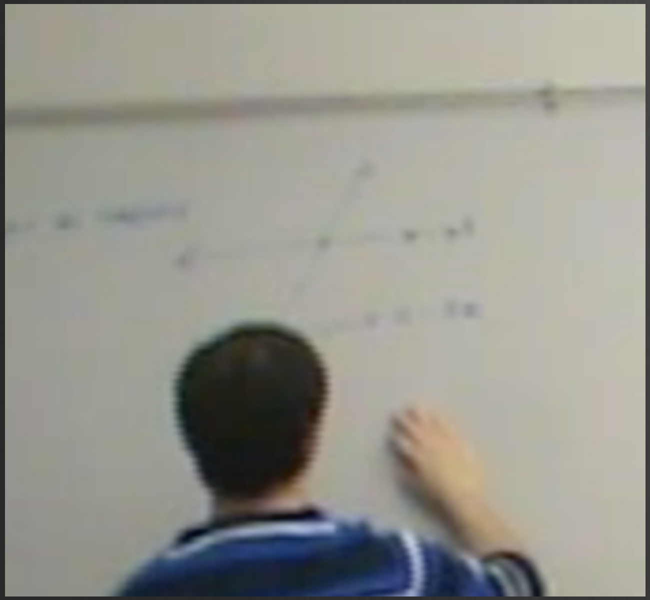
Students: Axioms.

I: We need a new axiom. We need a new axiom that says this and our standard geometry are both examples of what we currently have, but it is too general and we need to specify more.

Episode 2c: The square

S2: Hold on. We can construct this line that is parallel to that.

S1: But do you know that it passes through?



Episode 2c: The square

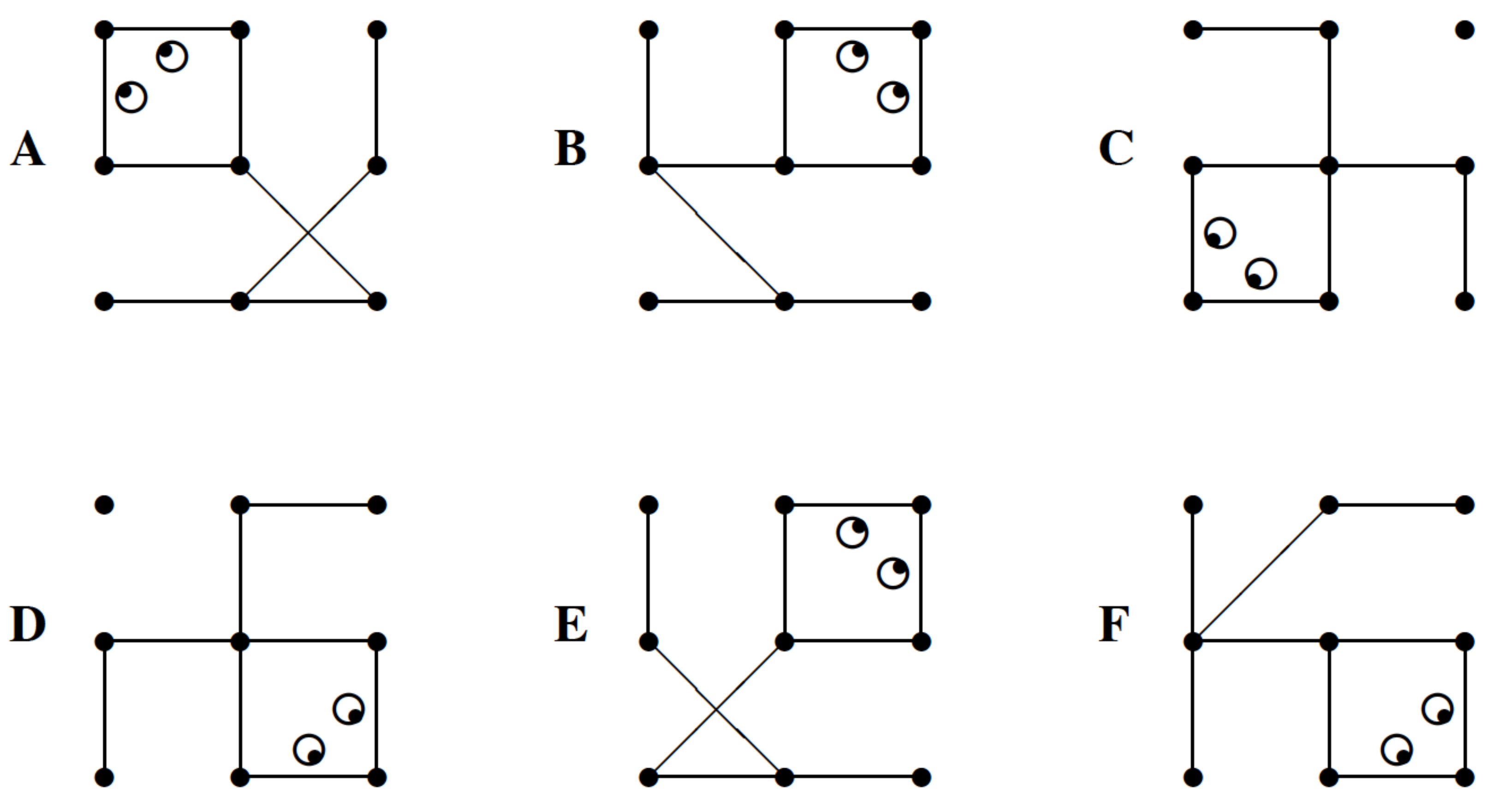
S2: That there is an intersection here?

S1: And there.

S2: Well, I’m saying it’s parallel at this point here. We know it has…

(pauses and looks for 45 seconds)

Episode 3: “Defining” Congruence



Episode 3: Congruence

I: How many of you are sure? (Some people raise hands) So how many things would you have to check to make sure that it does in fact actually match up perfectly on top of each other?

BC: Can you explain the question to us?

I: So we got an operation that takes the points of D and puts them somewhere and we feel like it matches perfectly on top of C, yes? We do know that the center of D ends up on the center of C right? But are we sure about any of the other points being in the right place?

BP: Not technically.

BC: Can we like trace the objects because we don't know if the lengths on object D are the same as object C.

I: OK, so we could do science to it instead of mathematics. We could try to say well OK I've measured this one really precisely and so I'm pretty confident but what's the problem with measurement? Grounding, precision, you can't be infinitely precise with a measurement so this is a problematic, I mean in practice it's a great way to go about doing it and if you were doing this with young ones that's what you would do, right?

Summary

- One social norm observed: *meta-argumentation*
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Corroboration: N_2

Students are asked to step back and consider *what* they might be able to know and *how.*

- S7: I really made sure that I took some of the things from this class to heart…you have a goal and you are trying to either prove something's true or false, or you show that you can't do what you were set up to do.
- S11: I always begin by kind of thinking about where I need to end up and how I'm gonna get there…[and] then try to say, "Oh! What do I know? What- what'd we learn before this? How can that help me?"
- Theorem 38: While we do not need angle measure to show that this theorem is true, the inclusion of the angle measures is for convincing, as we cannot create different swoops on the angles as we can within geogebra.

Corroboration: N_3

Students are asked to contrast different ways of knowing.

- 8M JL: Overall, though, I know math is not science, and so the senses are not enough to prove something is true or real, but at the moment, I'm struggling to answer the question "are mathematical objects real," which is a change from the beginning of the course when I would have answered this question, "Yes, of course."

- 8M ML: It's interesting how different disciplines approach truth. For example, there are psychologists whom believe that statements are true until proven false. Just because I study math, doesn't mean I follow mathematicians' means of defining universal truth. I'm still deciding whether I think about mathematical truth entirely through the lens of the mathematic discipline.

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Discussion Questions:

- What other meta-proving sociomathematical norms are there?
- Could these norms be seen outside geometry, perhaps in intro-to-proof courses?
- What pedagogical ways can a sociomathematical norm of proving be inserted into the classroom?
- How can we leverage our data to understand the development of these norms?

Thank you!

Further questions? Contact me at briankatz@augustana.edu

Corroboration: N_1

- N_1: Objectifying current and potential knowledge

- When describing the approach S11 took on a task prior to the interview:
    - "I made, like, examples of things that I thought were valid and things that I thought challenged what I started to believe in the first place."