Making Actions in the Proving Process Explicit, Visible, and "Reflectable"

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RUME 2010

Outline of Talk

- Need for a supplement in the beginning real analysis course
- Theoretical framework
- What we did and student difficulties
- Illustration of supplement proof
- Effect on the students

Need for a Supplement

- The real analysis course is a three credit junior level class which serves three purposes.
  - A beginning real analysis course for math majors.
  - An introduction to real analysis and proof for pre-service secondary math teachers.
  - Provides remediation for students beginning graduate students who are not succeeding in the beginning graduate real analysis course.
- In an interview with the real analysis teacher she mentioned the class tries to be all things to all students which is virtually impossible in three hours a week.

- When asked why she approached us to provide a supplement she said, “In my opinion students learn to do proofs by doing proofs and not reading them or doing exercises.” which can not always be done in the class setting.
- She went on to mention our previous course for improving graduate students proving abilities and thought the same thing would be helpful for the real analysis students.

Theoretical Framework

- We view the proving process as a sequence of actions both mental and physical.
- Some of these actions and the reasons behind them such as looking up a definition, drawing a sketch, or constructing an example are not visible in the final written proof.
- The actions in the proving process, when paired with a triggering situation, may become automated and can be considered lasting mental structures which we call *behavioral schema*.

(Selden, McKee & Selden, 2010)

- Our claim is that by making such actions, and the reasons for them, more visible, that through reflection and practice the students will be better able to carry out the appropriate actions autonomously.
- Further, by reflecting on those actions the link between certain situations and actions will be become strengthened and, as a result, turn into behavioral schemas.

- Finally, we suggest that by developing behavioral schemas which can ultimately be carried out easily, one's working memory is unburdened allowing one to concentrate on the problem-centered part of the proof.

(Selden, McKee, Selden, 2009)

What We Did and Student Difficulties

- We are currently in the third iteration of offering the supplement.
- The students who attend the supplement do so on a voluntary basis and every effort is made to conduct it at a time when almost every student in the real analysis course is able to attend.
- The supplement met once a week for 75 minutes, a total of one-third of the class time for those students who chose to attend.

- The teacher of the real analysis class choose a homework problem each week to be graded very carefully.
- The supplemental teachers worked the problem very carefully noting the actions. Then they selected or wrote a theorem which used many of the same actions but which was not a template problem.
- The students that attended the supplemental course would co-construct the proof with guidance from the supplemental teachers.

- One of the teachers of the supplement would write the theorem on the board. Then the students, or teacher if need be, would offer suggestions about which actions to do next.
- For each suggested action such as writing down a definition or drawing a sketch, a student was asked to carry out the action on the board.
- The goal was for the students to reflect on what occurred and later perform these actions or similar actions autonomously.

- Every student was encouraged to participate in co-constructing the proof although not every student could carry out every action.
- Class discussion and questions were actively encouraged.
- A handout was given to the students at the end of the supplemental class that went through the proof and described the actions.
- We took the point of view of Yackel, Rasmussen, and King (2000) that “when the classroom norm is that of making sense of other student’s reasoning, class discussions often form the basis for the students to further their own mathematical development.”

- The supplement was videotaped and field notes were taken.
- The teachers of the supplement and the real analysis course would meet following each supplemental class to review what happened and plan for the next supplemental class.
- The real analysis teacher would also use any misconceptions or difficulties that occurred during the supplemental class to inform her instruction. Further, she would imitate the actions in her lectures to reinforce the supplemental instruction.

Student Difficulties

- Not turning the pages of their book or notes to find the appropriate definitions, theorems, etc.
- Unable to copy a definition accurately.
- Unable to make an appropriate sketch at the appropriate time in the proof.
- Starting with the hypothesis rather than looking at the conclusion to see what is to be proved.

Illustration of Supplement Proof

- Theorem from Supplement: Let $\{a_n\}$ and be $\{b_n\}$ sequences, both converging to P. If $\{c_n\}$ is the sequence given by $c_n=a_n$ when n is even and $c_n=b_n$ when n is odd, then $\{c_n\}$ converges to P.

- Theorem from Class: Show that $\{a_n\}$ converges to A if and only if $\{a_n-A\}$ converges to 0.

- Proof of Supplement Theorem:

  Let $\{a_n\}$ and $\{b_n\}$ be sequences and P be a number so that $\{a_n\}$ and $\{b_n\}$ converge to P. Suppose $\{c_n\}$ is the sequence given by $c_n=a_n$ when n is even and $c_n=b_n$ when n is odd.

  Let $\varepsilon>0$.

  As $\{a_n\}$ converges there exists an N_a such that for all $i> N_a$, $|a_i-P|<\varepsilon$.
  As $\{b_n\}$ converges there exists an N_b such that for all $j> N_a$, $|b_j-P|<\varepsilon$.
  Let $N=\max\{N_a, N_b\}$.

  Let $n>N$.

  Case 1: Suppose n is even. Then $|c_n-P|=|a_n-P|<\varepsilon$.

  Case 2: Suppose n is odd. Then $|c_n-P|=|b_n-P|<\varepsilon$.

  In either case $|c_n-P|<\varepsilon$.

Actions in the Proof

- Write the first line.
- Write the last line.
- Unpack the conclusion.
  - Write the appropriate definition on scratch work.
  - Change the notation to fit the problem.
- Set-up the proof leaving appropriate spaces.
- Find N.
- Recognize the cases.
- Complete the proof including any necessary algebra.

Effect on Students

- In describing the attempts of students who attended the supplement to produce a proof on their exam the real analysis teacher said "I would see the first line, I would see the last line… I can see the technique… some more obvious than others but most definitely it was on the test."
- As the semester went on the students knew what to do next with less prompting and help.

- In interviews with the students they all responded very positively to the supplement and what they learned.
- When asked how the supplement impacted how they do proofs in their current courses they replied:
  - Knowing where to start;
  - Knowing how to unpack the conclusion;
  - How to use the definitions;
  - How to use fixed but arbitrary.