Communicative Artifacts of Proof: Transitions from Ascertaining to Persuading

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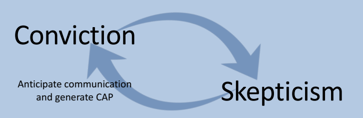
Ascertaining



Fosters emerging formality

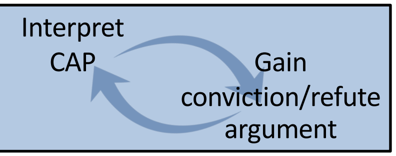
Presentation of CAP

Communicative Artifacts of Proof (CAP)

the observable products (verbal, written, gestural, etc.) of the process of proving or refuting a mathematical relationship or conjecture that an individual prover generates to communicate his or her reasoning about why the relationship does or does not hold to a community of proof.

Feedback informs future proof activity

Persuading



Community evaluation via normative standards

Proving as Problem Solving

There have been connections made between proving and problem solving (e.g., Weber, 2005). For example, Furinghetti and Morselli (2009) stated that "proof is considered as a special case of problem solving" (p. 71). Savic (2015) used a problem-solving framework (Carlson & Bloom, 2005) for the proving process of two participants, but found a small amount of actions that did not conform to the framework, including periods of incubation. However, Carlson & Bloom (2005) did discuss a perspective of problem-solving that dealt with skepticism and audience: the *monitoring* attribute. This attribute deals with both meta-cognition and consideration of aesthetics, which may inform the creation of the CAP. Finally, Carlson and Bloom's framework included checking, a validation phase that may involve skepticism.

Anticipating Communication

Individuals are likely to anticipate the communication of their ideas within a broader community. Anticipation would likely inform the ascertaining subprocess (particularly during moments of skepticism) and persuading subprocess during the development and presentation of the communicative artifacts of proof (CAP). Specifically, anticipation of communicating is informed by the individual's prior experiences within a given proof community and the individual's interpretation of the feedback provided by members in the community. Altogether, this fosters the emerging formality of students' proof activity.

Need to Discuss and Explore the Notion of CAP

The importance of the CAP is in acknowledging the distinctions between how individuals in a proof community construct mathematical meaning during communication. The prover sets out to convey his or her mathematical activity in the best way that he or she knows to communicate a convincing argument within a specific community. On the other side of this communication, the members in the proof community must individually re-construct the proof for themselves, interpreting the CAP from their own unique perspectives.

We do not view the CAP as containing any intrinsic meaning or validity. Rather, the prover and each individual member of the proof community ascribes meaning to the CAP for himself or herself. This is consistent with Weber's (2010) definition of student-centered proof, a perspective he motivates by saying, "…it is crucial to consider who is reading the proof; it is easy to imagine a proof that is explanatory to one student but not another and a good teacher cannot overlook this difference" (p. 34).

Our Theoretical Hypothesis on Ascertaining and Persuading

- The notion of CAP should inform the field's understanding and investigation of proof and the proving process by allowing proof researchers to distinguish between specific aspects of proof and focus on the specific proof activity in a participant's proving process
- This might allow researchers to pinpoint hardships that students experience in their proving process, or may allow students to specifically target self-evaluation of their own proving.
- These early notions of CAP can be developed to better explicate the types activity constituting the subprocesses of ascertaining and persuading.

As Researchers and Instructors

Researchers an instructors can leverage an attendance to the CAP along two aspects of focus as an audience: (1) interpreting the structure of the proof – formality and style of arguments that experts usually focus on when studying proof and (2) the mathematics of the prover that inform the development of such a proof (Plaxco, 2011; 2012).

This affords a lens that emphasizes the models of student understanding – of both proof and mathematical content – which informs research and assessment. It also allows researchers and instructors to situate a student's proof activity relative to his or her conceptual understanding, incorporating vital context for proof activity.

Proofs and Refutations

We view the affordances the CAP provides researchers and instructors as exemplified by Lakatos' (1976) "Proofs and Refutations." Specifically, example-oriented responses to proofs (local counterexamples and global counterexamples) reflect the proof community's focus on the prover's conceptual understanding, especially aspects of the prover's conceptual understanding that might not account for such counterexamples. The prover's response to such counter examples might then inform his or her proof approach or conceptual understanding. By attending to the CAP, researchers are able to focus on how each participant in a proof community is interpreting the other's communicative artifacts.

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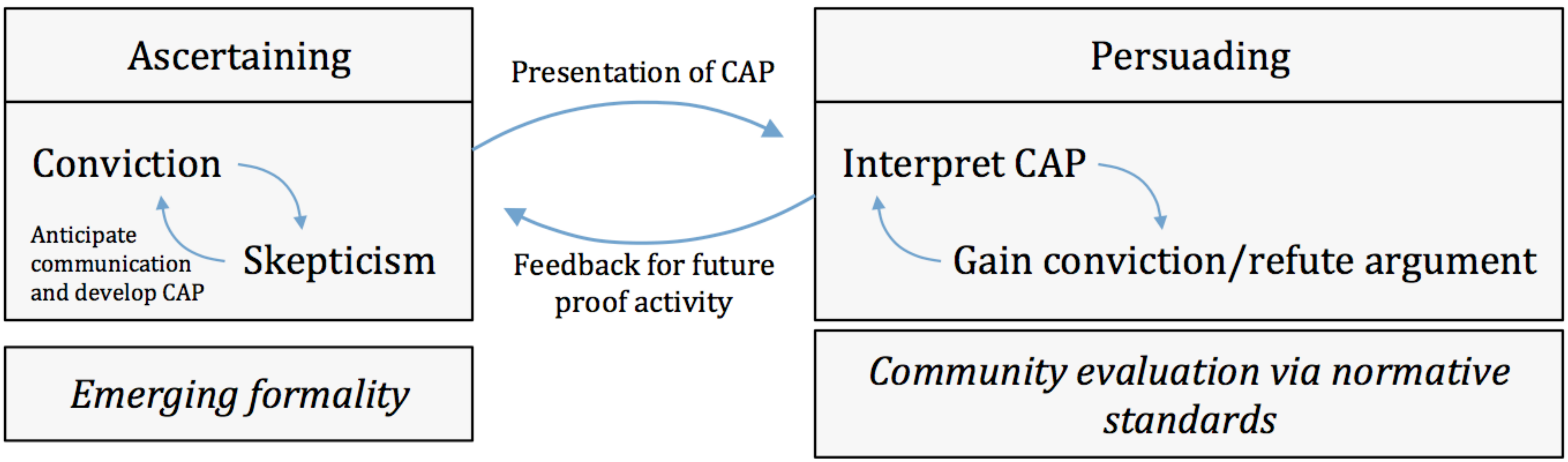
Ascertaining
Conviction
Anticipate communication and develop CAP
Skepticism
Emerging formality
Presentation of CAP
Feedback for future proof activity
Persuading
Interpret CAP
Gain conviction/refute argument
Community evaluation via normative standards