Pedagogical Practices for Fostering Mathematical Creativity in Proof-Based Courses: Three Case Studies

Some mathematics education publications highlight the importance of fostering students' mathematical creativity in the undergraduate classroom. However, not many describe explicit instructional methodologies to accomplish this task. The authors attempted to address this gap using a formative assessment tool named the Creativity-in-Progress Rubric (CPR) on Proving. This tool was developed to encourage students to engage in practices that research studies, mathematicians, and students themselves suggest may promote creativity in processes of proving. Three instructors in different institutions used a variety of tasks, assignments, and in-class discussions in their proof-based courses centered around the CPR on Proving to explicitly discuss and foster mathematical creativity. These instructors' actions are explored using Levenson's four teacher roles of fostering mathematical creativity. In this report, preliminary results indicate that each of the three instructors assumed at least three of the four roles.

Key words: Mathematical creativity, teaching practices, proof-based courses

The importance of mathematical creativity and fostering it in mathematics courses has been discussed in many mathematics education publications, including policy documents. Most recently, mathematical creativity has been emphasized by the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics in its latest guidelines (Schumacher & Siegel, 2015). The guidelines state that "[a] successful major offers a program of courses to gradually and intentionally leads students from basic to advanced levels of critical and analytical thinking, while encouraging creativity and excitement about mathematics" (p. 9). Under the subheading *Cognitive Goals and Recommendations*, the guidelines also state "these major programs should include activities designed to promote students' progress in learning to approach mathematical problems with curiosity and creativity and persist in the face of difficulties" (p. 10). Nadjafikhah, Yaftian, and Bakhshalizadeh (2012) provide an even stronger claim: one of the goals of any educational system should be to foster mathematical creativity. However, as much as mathematical creativity is discussed as an important aspect in undergraduate mathematics (e.g., Zazkis & Holton, 2009), pedagogical actions that support its explicit fostering in classrooms are rarely mentioned or studied. Ervynck (1991) stated, "[W]e therefore see mathematical creativity, so totally neglected in current undergraduate mathematics courses, as a worthy focus of more attention in the teaching of advanced mathematics in the future" (p. 53). In this project, the authors attempt to explore this issue further by attempting to address the research question: ***What teacher actions or practices in the proof-based undergraduate classroom might foster students' perceptions of mathematical creativity?***

To explicate mathematical creativity with undergraduate students, three instructors from different universities in the U.S. implemented various practices such as designing assignments, creating tasks, and structuring class discussions in their courses. One common feature of these practices was that all three instructors centered their implementations around a tool created to enhance research-based actions for mathematical creativity in the proving process, the Creativity-in-Progress Rubric (CPR) on Proving (Savic et al., 2016; Karakok et al., 2016). This rubric was developed considering certain theoretical aspects of mathematical creativity, which is discussed in the following section.

Theoretical Perspective and Background Literature

There are over 100 different definitions of mathematical creativity (Mann, 2006) and multiple theoretical perspectives (Kozbelt, Beghetto, & Runco, 2010). In developing the CPR on Proving, the authors considered mathematical creativity as a *process* that involves different modes of thinking (Balka, 1974) rather than looking at the creative end-product (Runco & Jaeger, 2012). Mathematical creativity in the classroom often considers a *relative perspective*, similar to Liljedahl and Sriraman's (2006) description of mathematical creativity at the K-12 level: a process of offering new solutions or insights that are unexpected for the student, with respect to his/her mathematics background or the problems s/he has seen before. This particular definition acknowledges students' potential for creativity (both in process and product) in the mathematics classroom. Often literature cites this as "little-c" creativity (Beghetto, Kaufman, & Baxter, 2011), as opposed to "big-C" or an absolute perspective (Feldman, Csikszentmihalyi, & Gardner, 1994). Finally, the authors focus on creativity in the domain of mathematics, instead of exploring creative endeavors in general (Torrance, 1966). Many researchers (e.g., Baer, 1998; Milgram, Livne, Kaufman, & Baer, 2005) also stressed this distinction and the importance of domain-specific creativity: "creativity is not only domain-specific, but that it is necessary to define specific ability differences within domains" (Plucker & Zabelina, 2009, p. 6).

Creativity-in-Progress Rubric (CPR) on Proving

The CPR on Proving was rigorously constructed through triangulating research-based rubrics (Rhodes, 2008; Leikin, 2009), existing theoretical frameworks and related studies (Silver, 1997), conducting studies exploring mathematicians' and students' views on mathematical creativity (Tang et al., 2015), and investigating students' proving attempts (Savic et al., 2016).

There are two categories of actions that may help a student foster mathematical creativity: *Making Connections* and *Taking Risks*[1]. Making Connections is defined as the ability to connect the proving task with definitions, theorems, multiple representations, or examples from the current course that a student is in or possible prior course experiences. Taking Risks is defined as the ability to actively attempt a proof, demonstrate flexibility in using multiple approaches or techniques, posing questions about reasoning within the attempts, and evaluating those attempts. Making connections has three subcategories (between definitions/theorems, between examples, and between representations), and Taking Risks has four subcategories (tools and tricks, flexibility, posing questions, and evaluation of a proof attempt) that are designed to have students explicitly think about ways to develop aspects of their own mathematically creative processes.

Teaching for Development of Creativity

The literature for teachers' actions to develop mathematical creativity at the undergraduate level is scarce. Zazkis and Holton (2009) cite a few implicit instances or strategies for encouraging mathematical creativity, including learner-generated examples (Watson & Mason, 2005) and counterexamples (Koichu, 2008), multiple solutions/proofs (Leikin, 2007), and changing parameters of a mathematical situation (Brown & Walter, 1983). From the K-12 literature, there are quite a number of articles of fostering mathematical creativity through problem posing (e.g., Silver, 1997; Knuth, 2002) or open-ended problems (e.g., Kwon, Park, & Park, 2006). However, there is still a need to understand what teacher actions in the classroom could foster mathematical creativity, especially in undergraduate mathematics courses.

[1] For a final version of the CPR on Proving, see Karakok et al. (2016).

Methods

Participants

Three different instructors, Drs. Eme, X and Omar, from three different institutions (two located in Western US and one located in Northeastern US) participated in this study. Each instructor used the CPR in Proving, although Drs. Eme and X removed the word "creativity" from the rubric in an attempt to minimize the explicit influence of the rubric on students' development of ideas on mathematical creativity. Both Drs. Eme and X implemented IBL teaching pedagogies whereas Dr. Omar also included lectures in his course. Dr. Eme implemented the CPR on Proving in her Transition to Advanced Mathematics course in Spring 2016 semester. She introduced it mid-semester. Dr. X's implementation was in a seminar on Elementary Number Theory during Fall 2015 where he introduced the rubric in the third week. Dr. Omar implemented the CPR in his Combinatorics course in Spring 2016 and used it in Portfolio Assignments.

Data

Prior to the start of the semester, all instructors discussed their course goals with the researchers and shared their CPR on Proving implementation plans with the researchers. Drs. Eme and X were involved in the development of the CPR on Proving where as Dr. Omar approached the authors to utilize the CPR on Proving in a course that he was designing. All three instructors met with the authors regularly to discuss the process of their utilization of the CPR on Proving throughout the semester. All three instructors collected their students' work and utilization of the CPR on tasks. Dr. Eme also audio-recorded in-class sessions. Students in the three courses were invited to participate in interviews at the end of the semester. In this preliminary report, we share our analysis of notes from instructors' self-reported actions in class and implementation plans, along with recorded implementations of the CPR in their courses.

Three instructors had different ways to introduce the idea of mathematical creativity and implementation of the CPR on Proving. For example, two months into the course, Dr. Eme started a class period giving students some of their own solutions to their exams: "...That's the exam 2 'solutions' and I say solutions in quotes because they're not all 100% correct, okay, but it doesn't matter. You know there are still really good ideas in there and that's what I want you to see." Dr. Eme then used the CPR in the discussion later in the same class period by placing written scratch-work on the document camera that was produced by a former student in a course that Dr. Eme taught in Spring 2015 semester. The theorem proved by this student was, "If $3|n$, then n is a trapezoidal number (a number that can be decomposed into a sum of two or more consecutive integers)" The discussion below ensues:

1 Dr. Eme: What did you guys get for the first one?
2 Stephanie: Advancing
3 Dr. Eme: Advancing? Why?
4 Stephanie: Because they were able to utilize multiple theorems and
5 definitions…Definition Q, the consecutive integers, Definition test 3.
6 Dr. Eme: Good. Good. Other people agree? Disagree?
7 Tony: Agree.
8 Dr. Eme: Agree? Ok. How about "between representations?"
…

9 Cargo: That the between representations still confuses because I'm not sure exactly that it means? Is it supposed to be like using the notation or what?
10 Dr. Eme: Yeah, that's a good question. Does anybody have an answer?
11 Stephanie: I would say it's anyway you can rewrite it, or draw a picture, or anything you
12 can do to represent that same concept but in a different way.
13 Cargo: OK.
14 Dr. Eme: That helps?
15 Cargo: Yeah.
16 Penny: That concept being.... the if $3|n$, let be trapezoidal number or any like any
17 definition?

Dr. X asked the students to use the CPR on five occasions throughout the semester, during homework as an evaluative tool, and during the final exam as extra credit. For example, in a homework problem, a student provided some scratch-work in his proof, bracketed the scratch-work, and wrote the reason why this scratch-work was not leading to a correct proof (in the student's words, "a mistake"). The student then promptly proved the theorem, utilizing the evaluation subcategory of the CPR on Proving.

Finally, Dr. Omar utilized the rubric while handing out problems labelled as "portfolio problems," which are, quoting from the syllabus, "much more involved, and the intention is to allow freedom to roam with it in any direction you wish." The students were required to use the rubric in a minimum three-page write up summarizing the proving processes they used. Unbeknownst to the students, many of these portfolio problems were open in mathematics, and the one portfolio problem had the same weight as the other three problems in the assignment which Dr. Omar viewed as "exercises". Dr. Omar stated that these three additional problems could be done by directly implementing ideas from class lectures or discussions.

Analytical Framework

To explore these three instructors' teaching actions, the authors adapted the work of Levenson (2011, 2013), who investigated fifth- and sixth-grade classes with the intention of explicating collective creativity and its effects on an individual's mathematical creativity. She described four teacher roles in fostering mathematical creativity:

1. choosing appropriate tasks,
2. fostering a safe environment where students can challenge norms without fear of repercussion,
3. playing the role of expert participant by providing a breakdown of the mathematics behind a process, and
4. setting the pace, allowing for incubation periods. (Levenson, 2013, p. 273)

The authors conducted preliminary analysis on instructors' actions during implementation of the CPR on Proving in their courses using these four roles. In particular, this preliminary analysis focused on how the CPR on Proving was utilized to foster mathematical creativity in their classroom, and using student interview data to support that fostering occurred.

Preliminary Analysis Results

The authors observed that all three instructors reported utilizing tasks that would encourage the mathematical creativity (property 1). For example, all three instructors choose tasks that were not all "Type 1" tasks, i.e. "proofs…can depend on a previous result in the notes" (Selden &

Selden, 2013, p. 320), but rather either “Type 2:” “require formulating and proving a lemma not in the notes, but one that is relatively easy to notice, formulate, and prove” (p. 320) or “Type 3,” which is hard to notice a lemma needed. Moreover, Dr. Omar assigned open-ended tasks, and Drs. Eme and X had their students analyze solutions or proof processes of others as part of classroom discussions, which we view as an action related to choosing appropriate tasks since students were required to think analytically about both a solution and proving process.

The authors noticed from Dr. Eme’s classroom data that this instructor explicitly tried to foster a safe environment (property 2 of Levenson’s framework). For example, as seen in the introduction episode that is shared, she carefully stated that the “solutions” the instructor was sharing out were not correct, but contained ideas that were “still really good.” This particular teacher action challenged the common norm of “only correct solutions should or would be valued” in this course. In addition, this action may also allow the students to challenge norms themselves without repercussion, since their instructor modeled such an action.

In the dialogue above, the students looked at another student’s proof of the trapezoidal number theorem. The authors claim that both the theorem and the evaluation of a student’s proof using the rubric are two examples of choosing appropriate tasks (property 1) for fostering students’ perceptions of creativity. Also, Dr. Eme is setting the pace to allow for incubation periods (property 4) in the course by letting students wrestle with how to interpret the CPR on Proving by analyzing the student’s scratch-work (see lines 8-17).

Property 4 is more apparent in the “appropriate tasks” that Dr. Omar assigned during class, since he knew those tasks were open, and therefore necessitated incubation time. The tasks themselves are a form of property 1, since they had the necessary elements for mathematical creativity to occur (through the lens of CPR). That is not to say that mathematical creativity can only occur in open math problems; tasks that both Dr. Eme and X provided can also elicit relative mathematical creativity.

Finally, each of the four properties of teachers fostering mathematical creativity (Levenson, 2013), seemed to appear in different forms in these instructors’ implementations of the CPR on Proving. For example, even though Dr. Omar did not have regular in-class discussions about the CPR on Proving in class, his approach to assignments encouraged the students to experience his “role of expert participant” (property 3).

Discussion/Conclusion

Creativity in mathematics is important for both mathematicians doing mathematics and students' development of mathematical actions. Three instructors of this study used the CPR on Proving differently in their courses. However, all three had a shared goal: a recognition or awareness of students' own proving processes and the actions that could lead to the development and enhancement of students’ perceptions of mathematical creativity. Drs. Eme and X had explicit discussions related to rubric categories, which are important to “increase student learning, motivate students, support teachers in understanding and assessing student thinking, shift the mathematical authority from teacher (or textbook) to community” (Cirillo, 2013, p. 1). Discussions can lead to student reflections of their own work. Dr. Omar asked students to utilize the CPR on Proving to reflect on their portfolio problem assignments. The CPR on Proving seemed to facilitate the explicit valuing of such meta-cognitive practice. As Katz and Stupel (2015) stated, “Creative actions might benefit from meta-cognitive skills and vice versa, regarding the knowledge of one’s own cognition and the regulation of the creative process” (p. 69).

References

Baer, J. (1998). The case for domain specificity of creativity. *Creativity Research Journal, 11*(2), 173-177.

Balka, D. S. (1974). Creative ability in mathematics. *Arithmetic Teacher, 21*(7), 633-638.

Beghetto, R. A., Kaufman, J. C., & Baxter, J. (2011). Answering the unexpected questions: Exploring the relationship between students' creative self-efficacy and teacher ratings of creativity. *Psychology of Aesthetics, Creativity, and the Arts, 5*(4), 342-350.

Brown, S., & Walter, M. (1993). *The art of problem posing.* Philadelphia, PA, USA: Franklin Press.

Cirillo, M. (2013). *What does the research say the benefits of discussion in mathematics class are?* NCTM Discussion Research Brief. Retrieved from: http://www.blockfest.org/research-brief-19-benefit-of-discussion.pdf

Ervynck, G. (1991). Mathematical creativity. In D. Tall, *Advanced Mathematical Thinking* (pp. 42-52). New York, NY, USA: Kluwer Academic Publishers.

Feldman, D. H., Csikszentmihalyi, M., & Gardner, H. (1994). *Changing the world: A framework for the study of creativity.* Praeger Publishers/Greenwood Publishing Group.

Karakok, G., Savic, M., Tang, G., El Turkey, H., Plaxco, D., & Naccarato, E. (2016, Feb/Mar). A rubric for creativity in writing proofs. *MAA FOCUS, 36*(1), pp. 42-43.

Katz, S., & Stupel, M. (2015). Promoting creativity and self-efficacy of elementary students through a collaborative research task in mathematics: A case study. *Journal of Curriculum and Teaching, 4*(1), 68-82.

Knuth, E. (2002). Fostering mathematical curiosity. *The Mathematics Teacher, 95*(2), 126-130.

Koichu, B. (2008). If not, what yes? *International Journal of Mathematics Education in Science and Technology, 39*(4), 443-454.

Kozbelt, A., Beghetto, R. A., & Runco, M. A. (2010). Theories of creativity. In *The Cambridge Handbook of Creativity* (pp. 20-47). New York, NY, USA: Cambridge University Press.

Kwon, O. N., Park, J. H., & Park, J. S. (2006). Cultivating divergent thinking in mathematics through an open-ended approach. *Asia Pacific Education Review, 7*(1), 51-61.

Leikin, R. (2007). Habits of mind associated with advanced mathematical thinking and solution spaces of mathematical tasks. *The Fifth Conference of the European Society for Researchers in Mathematics Education - CERME 5*, (pp. 2330-2339).

Leikin, R. (2009). Exploring mathematical creativity using multiple solution tasks. In R. Leikin, A. Berman, & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 129-145). Haifa, Israel: Sense Publishers.

Levenson, E. (2011). Exploring collective mathematical creativity in elementary school. *The Journal of Creative Behavior , 45*(3), 215-234.

Levenson, E. (2013). Tasks that may occasion mathematical creativity: Teachers' choices. *Journal of Mathematics Teacher Education, 16*(4), 269-291.

Liljedahl, P., & Sriraman, B. (2006). Musings on mathematical creativity. *For the Learning of Mathematics*, 17-19.

Mann, E. (2006). Creativity: The essence of mathematics. *Journal for the Education of the Gifted, 30*(2), 236-260.

Milgram, R. M., Livne, N. L., Kaufman, J. C., & Baer, J. (2005). Creativity as a general and a domain-specific ability: The domain of mathematics as an exemplar. In J. C. Kaufman, &

J. Baer (Eds.), *Creativity Across Domains: Faces of the Muse* (pp. 187-204). Psychology Press.

Nadjafikhah, M., Yaftianb, N., & Bakhshalizadehc, S. (2012). Mathematical creativity: Some definitions and characteristics. *Procedia - Social and Behavioral Sciences, 31*, 285-291.

Plucker, J., & Zabelina, D. (2009). Creativity and interdisciplinarity: One creativity or many creativities? *ZDM, 41*, 5-11.

Rhodes, T. (2010). *Assessing Outcomes and Improving Achievement: Tips and Tools for Using Rubrics.* Washington, DC: Association of American Colleges and Universities.

Runco, M. A., & Jaeger, G. G. (2012). The standard definition of creativity. *Creativity Research Journal, 24*(1), 92-96.

Savic, M., Karakok, G., Tang, G., El Turkey, H., & Naccarato, E. (2016). Formative Assessment of Creativity in Undergraduate Mathematics: Using a Creativity-in-Progress Rubric (CPR) on Proving. In R. Leikin, & B. Sriraman, *Creativity and Giftedness: Interdisciplinary Perspectives from Mathematics and Beyond* (pp. 23-46). Germany: Springer.

Schumacher, C. S., & Siegel, M. J. (2015). *2015 CUPM Curriculum Guide to Majors in the Mathematical Sciences.* Washington, DC: Mathematical Association of America.

Selden, A., & Selden, J. (2013). Proof and problem solving at the university level. *The Mathematics Enthusiast, 10*(1&2), 303-334.

Silver, E. (1997). Fostering creativity through instruction rich in mathematical problem solving and posing. *ZDM Mathematical Education, 3*, 75-80.

Tang, G., El Turkey, H., Savic, M., & Karakok, G. (2015). Exploration of undergraduate students' and mathematicians' perspectives on creativity. In T. Fukawa-Connelly, N. Engelke Infante, K. Keene, & M. Zandieh (Ed.), *Proceedings of the 18th Annual Conference on Research in Undergraduate Mathematics Education* (pp. 993-1000). Pittsburgh, PA: Mathematical Association of America.

Torrance, E. P. (1988). The nature of creativity as manifest in its testing. In R. J. Sterberg, *The nature of creativity: Contemporary psychological perspectives* (pp. 43-75). New York, NY: Cambridge University Press.

Watson, A., & Mason, J. (2005). *Mathematics as a constructive activity: Learners generating examples.* Mahwah, NJ, USA: Lawrence Erlbaum Associates.

Zazkis, R., & Holton, D. (2009). Snapshots of creativity in undergraduate mathematics education. In R. Leikin, A. Berman, & B. Koichu, *Creativity in mathematics and the education of gifted students* (pp. 345-365). Rotterdam, the Netherlands: Sense.