Utilizing A Research-Based Rubric To Assess Students' Creativity In Proof And Proving

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Background Literature

- Mann (2005) stated, “in seeking to facilitate the development of talented young mathematicians, neglecting to recognize creativity may drive the creatively talented underground or, worse yet, cause them to give up the study of mathematics altogether” (p. 239)
- Over 100 definitions of creativity in mathematics (Mann, 2005)
- Studies of K-12 Creativity (e.g., Silver, 1997; Leikin & Lev, 2013)
- Sriraman (2005) K-12 Creativity vs. “Mathematics” Creativity
- Zazkis and Holton (2009) discussed creativity in undergraduate mathematics, posing problems in many different content areas

Motivation

- Savic (2012) studied nine mathematicians and five graduate students working on proof tasks without time constraint
  - Many mathematicians utilized an "incubation" (break) period when stuck
- According to Wallas (1929) (and stated differently by Hadamard (1945), Poincare (1946) and Polya (1957)), incubation is one of the four stages of creativity
  - Preparation, Incubation, Insight, and Verification
- Savic and Karakok (in preparation) interviewed six mathematicians and all agreed that creativity is an essential piece of mathematics and mathematical proof

Research Question

- If mathematical creativity is both enacted and valued by mathematicians, then how might mathematical creativity be enacted and valued by our undergraduate students in proof-based courses?
- Our group believes that mathematical creativity should be **explicitly** discussed with our students.
- Therefore, we have created a rubric to assess and discuss what creativity in proving might be.

Creativity in Proving Rubric

- Sriraman (2005): "(a) the process that results in unusual (novel) and/or insightful solution(s) to a given problem or analogous problems, and/or (b) the formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle requiring imagination" (p. 24)

- The Creativity in Proving Rubric (CPR) was motivated by past research:
  - AAC&U (Rhodes, 2010) created a creativity rubric on essay-writing for incoming and outgoing undergraduate students
  - Torrance (1966, 1988) created tests for mathematical creativity
  - Leikin (2009) created a "creativity in problem solving" rubric, influenced by Silver (1997)

Three Categories of CPR

- Taking Risks
  - The ability to approach a proof and demonstrating flexibility in using different approaches
- Making Connections
  - The ability to demonstrate links between multiple representations, ideas from the current course that the student is in, and possible prior knowledge from previous courses
- Posing Questions/Conjectures
  - The ability to state a mathematical question that is either pertinent to the proof or can be proven itself

29: Thm: $3 \mid n$ if and only if 3 divides the addition of the digits of n. Let $n \in \mathbb{Z}$

Let $n = a_m 10^m + a_{m-1} 10^{m-1} + \dots + a_1 10^1 + a_0 10^0$

Let $s = a_m + a_{m-1} + \dots + a_1 + a_0$

Then $n - s = 9a_1 + \dots + a_{m-1}(10^{m-1} - 1^{m-1}) + 9^m a_m$

$= a_m 9^m + a_{m-1} 9^{m} + \dots + a_1 9$

$= 3[a_m 3^{2m} + a_{m-1} 3^{2(m-1)} + \dots + a_1 3^1]$

Thus $3 \mid n-s$ by definition S.

Now suppose $3 \mid s$. Since $3 \mid n-s$ and $3 \mid s$, $3 \mid n-s+s$ by Thm 28. So $3 \mid n$.

30: Every integer $n > 1$ has a prime divisor.

Let $n > 1$ and suppose n has no prime divisor. Assume that every $k \in \mathbb{Z}$ such that $1 < k < n$ has a prime divisor.

1) If n is prime, then n has a prime divisor n by definition T.

2) If n is not prime, let d be a divisor of n such that $1 < d < n$. Then $p \mid d$ where p is a prime number. By def. S, $d = pa$ where $a \in \mathbb{Z}$ and $n = db$ where $b \in \mathbb{Z}$. So $n = p(ab)$ and thus $p \mid n$ so n has a prime divisor.

Assume $n > 1$ is the smallest $\mathbb{Z}$ w/o a p divisor

$9^2 = (3^2)^2 = 3^4$

$9^3 = (3^2)^3 = 3^6$

$3 \cdot 3^4 = 3^5$

$3(3^2 + 3^3) = (3^2)^2 + (3^2)^3$

$9^2 + 9^3 = 3$

Discussion

- What level did you assign to the proving session under each of the 3 categories? Why?
- Without mention of creativity, how would you have assessed or graded this student's proof? Is it different than above?

IBL Teaching Strategies with the CPR

- Setting up the environment in the classroom:
  - Discuss/demonstrate proving attempts
    - Emphasize that mistakes can help
  - Pose theorems that require multiple solutions
  - Assign conjecturing tasks in homework
- Use the CPR when students are demonstrating or presenting proofs in the classroom

Conclusion

- S15: This [course] is definitely more conducive to creativity than a traditional course structure in that I cannot turn my homework in Calc 4 and have the wrong answer written down and get credit for it. I think that's probably pretty telling that I'm talking about credit rather than the learning process. But as far as learning things go, there is not much creativity in the things that I've learned previously.
- S11: I would say absolutely so. Just – just you know the setting itself lends to creativity cause we're being forced to be creative (chuckles).

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Questions?

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