How Can Fostering Creativity Build Student Self-efficacy for Proving?

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Mathematical creativity has been emphasized as an essential part of mathematics, yet little research has been done to study the effects of fostering creativity in the undergraduate classroom. In this talk, we explore the connection between creativity in the classroom using Sriraman's (2005) five principles for fostering mathematical creativity and student self-efficacy for proving using Bandura's (1997) four sources of gaining self-efficacy. Using classroom observations, online student surveys, and student interviews we explore potential connections between classroom use of the five principles of creativity and increased student self-efficacy for proving, highlighting cases where the use of multiple principles provided students greater opportunities for building self-efficacy for proving. Several implications of these connections and suggestions for future research are discussed.

Keywords: mathematical creativity, discrete math, proving, self-efficacy

For over a century mathematics and mathematics education researchers have endeavored to better understand the role of creativity in mathematical thinking and problem solving (Mann, 2006; Haavold, 2016). However, there is little research showing the impact of creativity in mathematics pedagogy on other cognitive constructs, such as self-efficacy. By investigating how creativity is valued in the classroom in relation to student self-efficacy, we can better understand how instrumental creativity is in students' mathematical development. In this paper, we investigate how the use of creative "principles" in the classroom potentially create opportunities for building students' self-efficacy for proving.

1. Theoretical Framework

1.1 Mathematical Creativity in the Classroom

There is considerable variation when seeking definitions of mathematical creativity (Mann, 2006). In this project, we defined *mathematical creativity* as one's process of offering new solutions or insights that are unexpected for the student with respect to their mathematical background. This definition is based on Savic et al. (2017), influenced by the perspectives of Liljedahl and Sriraman (2006). One may further categorize this definition as relative to the individual (Beghetto & Kaufman, 2007), process-oriented (Pelczer & Rodriguez, 2011), and domain-specific (Baer, 1998) mathematical creativity.

To characterize the presence or role of mathematical creativity in the classroom, we are interested in observable actions (or non-actions) instructors use to foster creativity in the classroom. Sriraman (2005) conjectured five principles that can be "applied in the everyday classroom to maximize the potential for creativity in the classroom" (p. 26). The definitions of these principles are derived from the mathematical creativity literature along with mathematicians' experiences of creating and publishing their results.

1.1.1. The Gestalt Principle. The Gestalt principle involves instructors providing students opportunities in or out of class to "engage in suitably challenging problems over a protracted time period, thereby creating the opportunities for the discovery of an insight and to experience the euphoria of the 'Aha!' moment" (p. 26). The "Aha" moment is the crucial third step in Wallas' (1926) four-step creative process due to the chance for a solution.

1.1.2. The Aesthetic Principle. Sriraman stated that "mathematicians have often reported the aesthetic appeal of creating a 'beautiful' theorem" (p. 27). The aesthetic principle applies to instructors valuing solutions that utilize unusual proving techniques, come from diverse topics of mathematics, or make efficient or elegant solutions. The teacher explicitly promotes the aesthetics of students' mathematical processes or products.

1.1.3 The Free-Market Principle. By Sriraman's recommendation, "teachers should encourage students to take risks" (2005, p. 28). The free market principle involves creating a classroom environment that allows students to freely input ideas, thoughts, and solutions. This also includes teachers encouraging students to "defend their ideas upon scrutiny from their peers."

1.1.4 The Scholarly Principle The scholarly principle involves creating a classroom environment "in which students are encouraged to debate and question the validity of… approaches to problems…, be encouraged to generalize the problem and/or the solution, as well as pose a class of analogous problems" (p. 28). This principle is equated to fostering students as scholars and allowing them to build mathematics off one another.

1.1.5 The Uncertainty Principle. The uncertainty principle "requires that students be exposed to the uncertainty and the difficulty of creating mathematics" (p. 28). According to Sriraman's conjecture, the teacher must take actions "cultivating this trait of perseverance" (p. 28). This requires that instructors attend to frustrations and balance difficulty while also developing a tolerance for ambiguity.

In this paper, we are interested in studying the influence of these five principles on student's mathematical development. While several ways to measure student creativity (Leikin, 2009) and student experience of these principles (Haavold, 2016) have been offered, there are limitations to these instruments. For example, quantifying and comparing creativity between students may run counter to the goals of those seeking to foster individual creativity. To further emphasize the individualistic benefits of fostering creativity, we turn to self-efficacy as one unrelated construct that may have a large impact on the mathematical success of students.

1.2 Self-efficacy

Bandura (1997) defined perceived self-efficacy as "beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments" (pg. 3). Thus, self-efficacy is also domain-specific, that is, self-efficacy can vary depending both the task in question and context one is working in. In this paper, we study students' self-efficacy for proving mathematical statements.

Self-efficacy plays a large role in students' motivation, engagement, persistence, resilience to adversity, and how much stress and anxiety people experience in engaging a given task (Bandura, 1997). Studies have shown that students with high self-efficacy show increased motivation and use of strategic thinking, manage their time better, are more persistent, and are less likely to reject correct solutions (Bouffard-Bouchard, 1990, Bouffard-Bouchard et al., 1991). In other words, self-efficacy mediates how students process and manage cognitive resources (Pajares & Kranzler, 1995; Randhawa, Beamer, & Lundberg, 1993).

Thus, we are interested in better understanding *how* students can gain self-efficacy for proving through classroom experience. Bandura (1997) outlined four primary sources of self-efficacy information: enactive experiences, vicarious role-modeling, verbal persuasions, and physiological reactions. Although these sources do not directly influence self-efficacy, "a host of personal, social, and situational factors affect and direct how socially mediated experiences are cognitively interpreted" (p. 79). Therefore, it is through cognitive processing and reflective thought that these sources of self-efficacy, as described below, are selected, weighted, and integrated into self-efficacy judgements.

1.2.1. Enactive Experiences. Enactive experiences refer to one's own successes in accomplishing a given task. An example of an enactive experience could be one's experience of solving a difficult proof and successfully explaining it to someone else. According to Bandura (p. 8), enactive experiences are often the most powerful source of self-efficacy since one's own experiences provide a reliable indication of future ability.

1.2.2. Vicarious Role-modeling. Vicarious role-modeling involves observation of someone else's competencies through which, by self-comparison, the observer bases judgments of their own ability. Observing someone else present their own proof could provide some indication of the observer's ability to relate to, or see them themselves similar to, the presenter.

1.2.3. Verbal Persuasion. Verbal persuasion involves direct verbal appraisal of one's ability by someone else. Telling a student, "I believe you have the resources to prove this" can serve as some indication of ability, but depends on the credibility of the persuader, and is considered less reliable than previous two sources. Verbal persuasion is less influential, since beliefs that are described rather than observed, depend on the credibility of the persuader, and will only "bolster self-change if the positive appraisal is within realistic bounds" (p. 101).

1.2.4. Physiological Reactions. Physiological reactions can include feelings of strength and stamina, or physical or emotional stress or fatigue. Feeling well rested or comfortable in the classroom are indicators of ability, while feeling of stress or fatigue are "signs of vulnerability to dysfunction" (p. 106).

1.3 Research Question

With both the principles and sources of self-efficacy, we turn to our research questions: how does an instructor's use of Sriraman's five principles create opportunities for students to build self-efficacy for proving, and do they build self-efficacy as a consequence of valuing mathematical creativity in the classroom?

2. Methods

Data was collected in two Discrete Math courses utilizing inquiry-based teaching at a large Midwest Research 1 university. In both courses, classroom observations (section 2.1), online surveys (2.2), and student interviews (2.3) were collected to explore then connection between the Presence of the Five principles in the classroom and changes in student self-efficacy, as outlined in Figure 1: Methods.

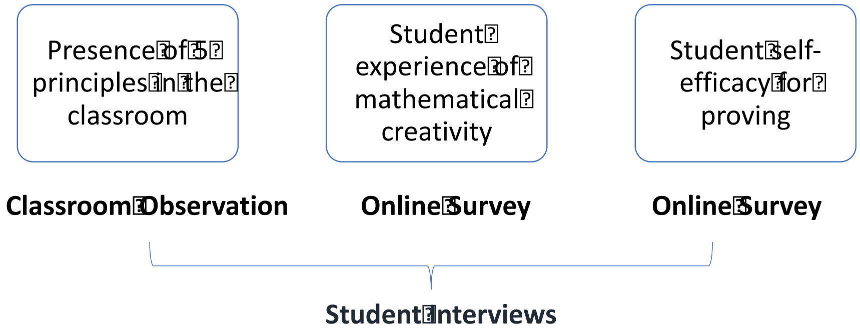

Figure 1: Methods of the study

The first course, an 8-week summer session taught by Dr. S, served as pilot data collection: five classes were videotaped, 8 students took online surveys, and 2 students were interviewed. Other than considering initial feedback on the online surveys, only the data from the student interviews from the summer semester will be discussed in this paper. In the fall semester, each class in the second course taught by Dr. F was videotaped, 22 students took pre/post online surveys, and 4 students were interviewed.

2.1 Classroom Observations

One class session from the fall semester was randomly chosen from the beginning (first five weeks), middle (weeks 6-10), and end (weeks 11-15) of the fall semester. Each session was viewed by both researchers and coded for evidence of instructor use of each of the five principles of creativity. Differences in codes were discussed until arriving at an agreement for each coded action. This served mainly to provide direct evidence of which principles, if any, were used in the classroom.

2.2 Online Surveys

The online surveys were designed to measure student experience of the five principles and self-efficacy for proving. The Five Principles survey consisted of ten questions, two per principle, asking students to rate how often they experienced each principle. This survey was given once at the beginning, rating their experience in their most recent math class, and a second time at the end of the current semester, rating their experience in that class. Figure 2: Five Principles Survey questions for the Gestalt Principle shows, as an example, the questions used for the Gestalt Principle in the pre-semester survey.

Instructions: Answer the following questions based on your experience in your **previous math class**.

1. How often did you have the freedom (of time and space) to work on a challenging problem or proof over a period two or more days? **2.** How often did you experience the joy of arriving at a solution after working on a problem or proof for several days?	(0) Never (1) 1 to 2 times per semester (2) 3 to 5 times per semester (3) 6-10 times per semester (4) Weekly (once or more per week, on average) (5) Daily (once or more per class period, on average)

Figure 2: Five Principles Survey questions for the Gestalt Principle

To construct a scale for measuring students' self-efficacy for proving – that is, their perceived capability for proving mathematical statements – we first searched the literature for existing self-efficacy scales for proving. Finding none, we followed Bandura's (2006) Guide to Constructing Self-Efficacy Scales to pilot such an instrument as follows:

First, to orient students to the domain of interest (mathematical proving), we begin by choosing mathematical tasks accessible to students entering a Discrete Math class. Next, to appropriately gauge gradation of challenge of the tasks, both researchers characterized each task on the continuum of problem difficulty offered by Selden & Selden (2013, pp. 303-305), shown in Table 1. Selden and Selden's (2013) continuum of problem difficulty below.

Table 1. Selden and Selden's (2013) continuum of problem difficulty

Type	Description	Example Theorem from Survey
1 – Very-routine	Can easily be proven from previous results	If n is an odd integer, then $n^4 - n$ is even.

2 – Moderately-routine	Requires formulating and proving a lemma (or trick) that is relatively easy to notice, formulate, and prove	The inequality $2^x \geq x + 1$ is true for every positive real number x.
3 – Non-routine	Requires formulating and proving a lemma (or trick) that is hard to notice, formulate, and prove	There does not exist a real number x for which $x^4 < x < x^2$.

Then, for each survey, three theorems, one of each of difficulty type, was selected. It was noted that these theorems were not included in any assigned homework, quiz, or exam.

To obtain a better overall measure of student's abilities related to the *process* of proving, and to provide context for students potentially unfamiliar with formal proving, we generated a series of subtasks based on the E-P spectrum for proving (Hsieh, Horng, & Shy (2012). They described proving "as the product of a spectrum of activities starting with exploration, and progressing to the stages of conjecturing, informal explanation, and justification" (pg. 288). From this, we arrived the following five subtasks:

1. Understand and informally explain why a statement is true or false.
2. Explore new ideas to come up with ways to start your proof.
3. Use various representations (numbers, pictures, tables, words) to structure your thinking
4. Formally write out and justify each step of your proof.
5. Examine your proof for accuracy and identify any missing steps.

Students were asked to rate their confidence (0%-100%) for each subtask of the three tasks. The students were also reminded that they were not required to actually prove the theorems. Three versions of this survey (each with distinct tasks) were given, at the beginning, middle, and end of the semester.

2.3 Student Interviews

Two students (pseudonymously named Sam and Scott) from the Summer session (taught by Dr. S) and four students (Fannie, Fred, Frank, and Francisca) from the Fall semester (taught by Dr. F) participated in a post-semester interview. Each student was asked questions about their classroom experience, their relative confidence in proving, and how they gained confidence in this class. Additionally, the students from the fall semester were given 30 minutes at the beginning of the interview to prove the same three tasks used for the end-of-semester self-efficacy survey and asked to describe their proving process. This was to attempt to further validate the self-efficacy survey, and due to page restrictions, will not be reported in this paper.

Each interview was transcribed, removing words such as "like" and "um" and "and stuff" for readability. Each interview was coded, once for explicit or implicit evidence of Dr. F's use of Sriraman's (2005) five principles for maximizing creativity, and again for evidence of Bandura's (1997) four sources of self-efficacy.

Codes were compared for inter-rater reliability and any discrepancies were discussed until a consensus was reached. Intersections between the five principles and self-efficacy codes were then analyzed. To illustrate this coding process, consider the following response:

Fred: When we would do the peer discussions in class, I would see how somebody else did it, and then I would be like "okay that makes a lot of sense, like how do you, kind of played around with, and how you got to where you went." And usually after class, I would have a break for six hours in between my next class. So, I would, a lot of times, go back and I can redo the whole, like two homework problems.

This was coded for explicit use of the scholarly principle due to the instructor's use of peer discussion in class in which students built off and evaluated one another's ideas, as well as implicit use of the Gestalt principle due to the course setup encouraging or giving students opportunities to rework problems a second time. However, while there is no mention of any of the four sources of self-efficacy, this response alone would not be considered in correlation with any principle and a change in Fred's self-efficacy for proving. Later, when asked, "what do you think contributed to your gaining confidence in proving?" Fred responded, "by becoming more creative" and "seeing how others went about being creative," explaining:

Fred: It helped me foster how I'm going build a way of thinking, of like, "okay if I'm trying to do this straight forward and if it doesn't work," like "alright, we gotta figure out some other way of doing it." And just like seeing how he and others would go about it; it really helped me in my learning process.

This segment was coded for implicit use the scholarly principle for engaging students in considering the validity of peer's approaches to problems, as well as for vicarious role-modeling, since Fred himself connected observation of someone else's competencies with his experience of gaining confidence for proving.

3. Results

Both classroom observations (3.1) and the online surveys (3.2) provided evidence of the use of the five principles in the fall semester's class. Additionally, the online surveys showed a definite increase in both students' experience of the five principles of creativity and their self-efficacy for proving. However, we did not have a large enough sample size (N=22) to draw any statistical correlations between classroom use of the principles and increased student self-efficacy for proving. Thus, we focus on the evidence from the student interviews (3.3) for connection between the five principles and increased self-efficacy.

3.1 Classroom Observations

In the first of the three class periods that were coded, all five principles were observed. At the beginning of class, Dr. F and the teaching assistant (TA) discussed how they were giving students an opportunity to redo some of the homework problems. The TA had given feedback to some of these problems which had been turned in online through a learning management system by providing open-ended responses to students' questions about their work, where they got stuck, etc. Several of these problems had been discussed the prior class period. Dr. F said that "we are giving you all a chance to redo some things, mainly because it is for you, not for us." This was coded as the explicit use of the Gestalt Principle for allowing students freedom of time and movement to foster "aha" experiences.

After this, Dr. F began responding a student question about "if-then" statements, asking, "'If x is an element of A, then x is an element of B.' Which [set theoretic] statement is that?" Students variously responded, "A union B," "A intersect B," and "A subset B." Without responding to the accuracy of any student responses, Dr. F said, "go to your notes." As students began agreeing "It's 'A subset B,'" Dr. F asked, "what is another truthful statement about this?" One student responded, "A equals B." Several students disagreed, which point Dr. F said, "Hold on, hold on, hold on. Why do you say, 'A equals B'?" This student commented, "Because if A is bigger than B, it's not contained in B. But if it's the same size as B, then they're equal," to which

Dr. F responded, “In fact, ‘A equals B’ can be one case. There [are] many cases, and one of them is ‘A equals B.’ What about some others with elements?”

The above interaction was coded for implicit uses of the uncertainty, scholarly, and free market principles. The uncertainty principle was coded for helping students become comfortable with open-ended problems by not responding to the accuracy of the answers; the scholarly principle was coded for engaging students in challenging the validity of their own responses; and the free market principle was coded for fostering a risk-taking environment by engaging a potentially wrong solution and using it to explain how their thoughts fit into the bigger picture.

This led to a conversation of contrapositive, inverse, and converse statements. At one point, Dr. F incorrectly stated, “this statement is the inverse,” to which a student corrected “[it’s the] converse, I just looked it up.” This was coded for implicit use of the scholarly principle, since this illustrated a norm running throughout the course; students had access to all the notes, were given authority to construct their own understanding from the notes and encouraged to refer to them in challenging Dr. F’s claims.

Toward the end of this discussion, Dr. F explained, “I believe this statement [if, then] is why we learn math. I believe that calculus, adding fractions, boils down to being logical. I believe this is why for 3500 years we have been learning math: to be more precise when we speak and talk.” This was coded for explicit use of the aesthetic principle. Dr. F was conveying the beauty, elegance, and precision of mathematical communication.

In the remainder of this class period, we coded one or more instances of each principle. In the second class, six instances of use of the free market and scholarly principles were coded, including several that came about as a result of one student, Fannie, leading the class discussion. In the third class, two instances of explicit use the free market principle were coded in relation to an assignment (worth 5% of their total grade) called “Productive Failure.” This assignment involved students presenting an experience where they failed in their proving process and explaining how it proved productive in the end. Dr. F responded to student questions about how to encourage a friend to present their productive failure saying, “I make mistakes in lecture, and am getting better at talking about my own failures. Having a difficulty and talking to someone else about it relieves the weight.” The counted instances of all the coding for the five principles for all three class periods are shown in Table 2.

Table 2. Frequency of coding of three randomly selected class periods

Principle	Beginning	Middle	End
Gestalt	2	1	1
Aesthetic	3	1	0
Free Market	3	6	3
Scholarly	3	6	2
Uncertainty	2	3	6

3.2 Online Surveys

As an initial measure of the internal consistency of the self-efficacy scales, the Cronbach’s alpha is 0.92, 0.90, and 0.92 for Surveys 1, 2, and 3, respectively ($\alpha \geq 0.9$ is excellent). The mean and standard deviation of the self-efficacy scores for each statement are shown in Figure 3: Mean and standard deviation of self-efficacy scores for each problem statement used.

Survey	Level	Statement	Mean	SD
1 (n=23)	1	If n is an odd integer, then n^2+1 is even.	64.09	15.64
	2	If a, b, c are positive integers, and ab, bc, and ac all have the same parity (are all even or all odd), then a, b, and c all have the same parity.	52.09	24.10
	3	If a and b are integers, then $a^2 - 4b \neq 2$.	57.57	21.88
2 (n=24)	1	If n is an odd integer, then $n^4 - n$ is even.	76.00	12.50
	2	The inequality $2^x \geq x+1$ is true for all positive real numbers x.	74.17	14.36
	3	There does not exist a real number x for which $x^4 < x < x^2$.	73.33	14.90
3 (n=21)	1	If $x, y \in \mathbf{R}$, then $\|x+y\| \leq \|x\| + \|y\|$.	84.67	10.13
	2	If n is an integer, then $1 + (-1)^n(2n-1)$ is a multiple of 4	81.43	13.22
	3	Every odd integer is the difference of two squares.	77.81	16.50

Figure 3: Mean and standard deviation of self-efficacy scores for each problem statement used

Initial comparison of pre- and post-semester for both five-principle ratings and self-efficacy scores shows a distinct difference. The T-test gave p-values of 3.63×10^{-5} and 8.97×10^{-7} for students' self-efficacy score and their average five-principle ratings (N=19). This difference can also be seen in Figure 4: Student self-efficacy score vs. average five-principle rating which plots each student's self-efficacy score vs. their average of all five principles ratings. The data from Fannie is highlighted in red, and described below.

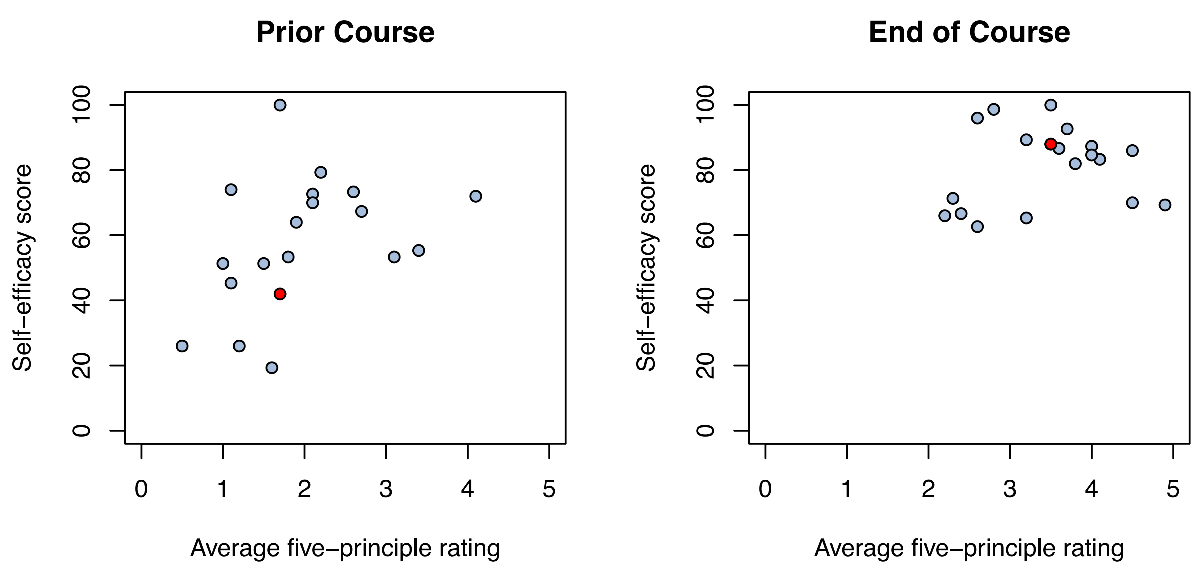

Figure 4: Student self-efficacy score vs. average five-principle rating

Fannie's self-efficacy scores from the beginning and end of the semester were 42 and 88, respectively. Her ratings of the five principles from her prior math course ranged from "1 to 3 times per semester" (for free market, scholarly, and uncertainty principles) to "3 to 5 times per semester" (for Gestalt and aesthetic principles), for an average of 1.7 (by the scale shown in

Figure 2: Five Principles Survey questions for the Gestalt Principle). Her ratings for the five principles in this course were "6-10 times per semester" (for free market and scholarly principles), "weekly" (for Gestalt and uncertainty principles) and "daily" (for the aesthetic principle), for an average of 3.5.

3.3 Student Interviews

Figure 5: Summary of connections found between the five principles and sources of self-efficacy provides a summary of the connections between the five principles and sources of student self-efficacy coded in the student interviews. Although most of these connections serve as positive sources of self-efficacy information, we will note one case in which the student experienced a negative source of self-efficacy (3.3.2). Throughout this section, interview quotes coded for **the five principles** are in **bold**, and *sources of self-efficacy* are *italic*. Each subsection provides support for the connections (See figure 5 for the exact subsection).

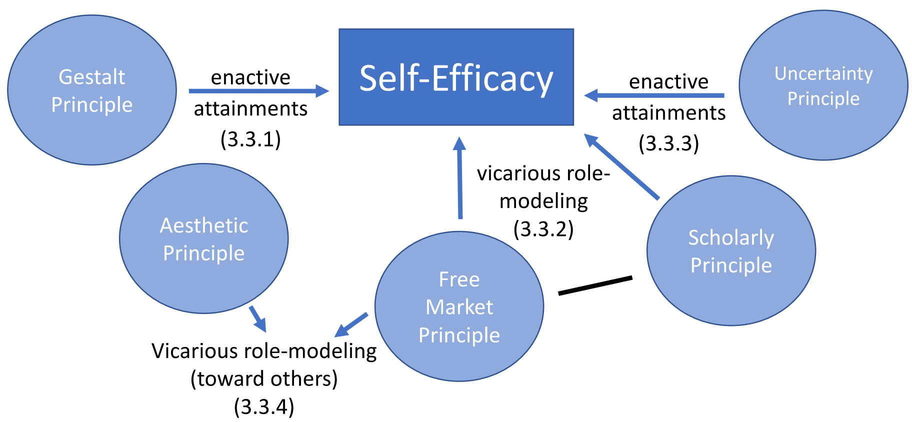

Figure 5: Summary of connections found between the five principles and sources of self-efficacy

3.3.1. Gestalt Principle and Enactive Attainments Each of the four students from the Fall semester described ways in which the Gestalt principle provided a positive source of self-efficacy through enactive attainments. Fred's experience in the course overall seemed to contribute to his self-efficacy:

> *Fred*: It's a roller coaster of a class. You reach points where you're so frustrated that you can't solve stuff, and then the satisfaction when you actually…**You figure out how to do a proof that you've been working on for a while.** There's really no more empowering feeling in the world…*You feel like you can do anything!* But yeah, it's the trials and tribulations. You'll struggle and then it's *figuring out how to use that struggle to achieve something*, in the future, using what you know doesn't work and like, "all right, this doesn't work. Let's try to think of something new that might work better."

This description shows evidence of "aha" experiences – the satisfaction of "figuring out how to do a proof that you've been working on for a while" – which we coded for implicit use of the Gestalt principle. At the very least, the instructor was assigning problems that allowed or required this approach to proving. This segment was also coded for enactive experiences because Fred was empowered by his success in proving, and even considered remembering that experience in the face of future difficulties. Thus, struggling on difficult problems and eventually proving them contributed to Fred feeling that he could "do anything," serving as a source of positive self-efficacy.

3.3.2. The Scholarly/Free Market Principle and Vicarious Influences Sam, one of the students form the summer session, described an increase in his confidence in proving from enactive experiences related to the Gestalt principle and vicarious role-modeling from the teacher as sources of self-efficacy; however, he described vicarious role-modeling from his peers as a negative source of self-efficacy:

Sam: A lot of times [Dr. S] would **introduce a new problem and tell us to work on it**. It'd be a completely brand-new problem, which I guess is good to try to be able to think of how you'd approach like a brand new concept… There [were] times when he would **engage the class like earlier on in the semester** and I felt comfortable about like speaking up and answering occasionally, but a lot of the time I didn't feel comfortable around my peers to like answer questions.

Interviewer: Do you think your confidence of varied depending on the subject, or how did you become more confident by the end? Because you said you were confident?

Sam: Okay well, I don't know. It's not my personality really to engage in class that much. I don't know. But in this class setting I felt like there were people in this class that already knew, like there's like **two people in particular, that would always answer all the questions** and … I just deferred the questions to them, so if the teacher posed a question to the class and they didn't answer it, then *I felt it like "well, I definitely can't answer it if they can't*."

This was coded for explicit use of the scholarly principle because the instructor was posing problems and giving students opportunities to contribute to and extend the classroom community's body of knowledge. However, Sam feeling like "*I definitely can't answer it if they can't,"* was coded as a negative source of self-efficacy. We also noted that throughout Sam's interview the free market principle was not coded: he did not cite any way in which the instructor encouraged risk taking or provided an environment where the student felt safe to take risks.

In the fall semester, all four students described teacher actions for the scholarly associated with positive sources of self-efficacy via vicarious role-modeling. For example, the following quote was both coded for the **scholarly principle** and *vicarious role-modeling*:

Fred: **When we would do the peer discussions in class, I would see how somebody else did it, and then I would be like, "okay that makes a lot of sense, how do you played around with it, and how you got to where you went**" … I would, a lot of times, go back and … redo the two homework problems. *And thinking of how the other person solved it, and then that really helped me foster ways of being more creative, as I've said, like using other people, like how other people work are creative, as a stepping stone for how I could be more creative.*

Additionally, two of the students from the fall semester, Fannie and Francisca described ways in which the free market principle, along with the scholarly principle, allowed for greater opportunities for gaining self-efficacy via vicarious role modeling.

Interviewer: What in class contributed to your building confidence?

Fannie: **The general environment of everyone not being afraid to fail. Just generally understanding that my peers weren't going to judge me for doing something wrong** was really refreshing. And definitely **having that time to work with other people was really important, because everyone kind of had their own perspective or their own different take on the problem**… *Someone next to you might have had like a different idea about it that's just as correct as yours*.

This was coded for implicit use of the free market principle because Fannie knew "her peers weren't going to judge her." It was also coded for the scholarly principle due to Dr. F allowing students to engage and understand other's approaches to problems. Finally, because Fannie attributed her gaining confidence to being able to work with others without fear of judgment, this was coded for vicarious role-modeling.

Fannie also described the importance of "hearing other's thought processes," and using one another's "individual strengths to come together to understand this problem and like make this proof", both coded as scholarly principle. Immediately following this, she described an implicit use of the free market principle through the way she experienced the environment of the class.

Fannie: I also liked that there **wasn't any like super overpowering voices in the class, because I think that might have just been a characteristic of the people in the class, or it might have been the environment**… I'm not really sure. But I know that I get super intimidated when there's just one person that's constantly dominating the conversation and I think that would have made me much more hesitant to speak up or present my proofs. So that was kind of nice: really understanding from day one that no one was going to the judge you for failure, that was a really important part of the class.

Her speaking up in class is evidence of her gaining self-efficacy to present her proofs in this context. In fact, in the second class-session we coded, she was offered to lead the classroom and eagerly took this opportunity discussion saying, "oh this is so exciting." Her experience provides evidence how the influence of the scholarly principle on increasing students' self-efficacy via vicarious role-modeling may be related to the use of the free market principle in the fall class.

3.3.3 The Uncertainty Principle and Enactive Attainments Three students from the fall semester described teacher actions for the uncertainty principle that was coded in association with increased self-efficacy via enactive attainment.

Interviewer: How did the environment influence your learning to proof and you're gaining confidence in class?

Frank: Confidence? **[The fall semester course] helped me see where a lot of pitfalls were, *and be okay with that***, *but also learn to anticipate those.* The ability to anticipate those was something that was pretty valuable I think.

Being helped to see to see where his mistakes in proving were, and be fine with them, was coded for implicit use of the uncertainty principle since Frank connected the classroom environment with ambiguity and uncertainty in the proving process. Additionally, his learning and being able to anticipate his mistakes was coded for enactive experiences.

Francisca described being challenged "every single step of the way, [it] challenged your thinking and how you approached math." Then the interview shifted to how that challenge influenced self-efficacy.

Interviewer: How do you think that contributed to your confidence in learning to prove?

Francisca: At first, it was nerve-racking cuz I wasn't getting things right, and I wasn't understanding things. But over the semester and over time, I actually talked to a couple of people about this: it was like, **"you don't have to be right in this class, cuz no one's gonna be right." There's like no concept of being correct**, and once you take away the idea of being correct or being right, *it makes your confidence level go up a lot more, cuz you're like, "I know that I did this and this is what I accomplished, and so I should be proud of the work that I've accomplished."*

Not having to be "right" and not considering what is the one "correct" way of doing things demonstrates implicit use of the uncertainty principle. Interestingly though, at least part of her realizing this came from her peers, evidence of vicarious role modeling. This quote also shows how this perspective helped her reframe her own perspective of her accomplishments. In this way, this experienced served as a potential source of self-efficacy, not necessarily in the information directly, but in the influence she gives to these or future accomplishments in making self-efficacy judgements.

3.3.4 The Principles and Vicarious Influences Toward Others Although we initially set out to code the interviews for sources of self-efficacy for students, we noted two specific cases in which the principles fostered ways in which the students provided a source of self-efficacy to other students not in the course.

Interviewer: Is there anything else you gained from class?

Fannie: I don't know. The ability to annoy my friends with math concepts. I was studying for my physics test the other day, and I went up to my friend, and was like "this is so cool" and it was one of the problems from my last test. I was like, **"you've got to hear this. There's these things called trapezoid numbers, and they're so cool."** And *I wrote it out on the chalk board*, and they're like, "okay." I'm like, **"It's cool. Numbers are cool!"** But, I don't know. *I did gain a lot of confidence*. Ultimately that was the biggest thing. Because you know, at the beginning I was like "eh, I don't know." But, towards the end, *I was like, "I can prove things. I can do it!"*

This was coded for the implicit use of aesthetic principle because Fannie gained an appreciation of something new (trapezoid numbers) and was compelled to explain it to her friends with confidence, becoming a potential source of self-efficacy to her friends.

Interviewer: How long did you spend on homework?

Francisca: So much time. I was like "oh it's a [sophomore level] course. It won't be…" Oh my God, **so much homework, so much time. I would spend like hours. My roommates would come home and I would be doing a problem, they'd go back to class and come back, and I'd still be doing the same problem.** And they're like "why? We've been gone for two, three hours and you're doing the exact same thing." …They also thought everything that I talked about for the whole semester was just absolutely crazy. I would bring up all the terms that we would use in class like **"productive failure"** and all the other things, and they're just like "you're nuts." *I was like, "no no no."*

This showed that Francisca's experience of the Gestalt and free market principles may have contributed toward her speaking out to her roommates, even in the face of rejection of her ideas. This also demonstrated how the principles may have encouraged Francisca to became a potential source of vicarious role-modeling toward others.

4. Discussion, Limitations, and Future Research

In this section, we combine the discussion of the results, limitations, and directions of future work based on findings from the classroom observations, online surveys, and student interviews.

4.1 Classroom observations

The classroom observations helped corroborate the use of the principles in the classroom with students self-reported experience from the online surveys and interviews. For future large-scale studies, due to the time investment necessary for recording and coding individual class periods, we suggest developing a survey similar to the one we used to measure students' experience of the five principles, to measure students' perception of how often their instructor utilizes the principles. Following the recommendation of Hayward, Weston, and Laursen (2018), such an instrument, if aligned with classroom observation, may more efficiently measure classroom presence of the five principles more than classroom observations.

4.2 Online Surveys

So far, our self-efficacy survey seems to be a reliable measurement of student self-efficacy for proving. The Cronbach alpha provides evidence that the self-efficacy scale is measuring one construct. For survey 2 and 3, the decreasing mean for more difficult problems gives some indication of construct validity since more difficult problems should correspond to lower self-efficacy. The lower mean and higher standard deviation of problem statement 2 on survey 1 may be due the statement being lengthy, so students were less able to quickly gauge their ability. Also, the higher standard deviation for survey 1 may have simply been a result of being less familiar with proving.

For future surveys, we recommend using the tasks from survey 2 and 3 for pre- and post-semester surveys to measure the change in student self-efficacy. This change, along with the modifications suggested in 4.1, would allow these surveys to be easily given on a larger scale, which is necessary for more accurately determining the correlation between these principles and changes in student self-efficacy for proving.

4.3 Student interviews

In the summer session, although students experienced the scholarly principle in association with vicarious role modeling, Sam experienced vicarious role-modeling as a negative source of self-efficacy. This appears to be similar to the phenomena of observed by Bandura (1997) that "observing others perceived to be similarly competent fail lowers observers' judgment of their own capabilities and undermines their effort" (p. 87). It may be that without adequate presence of the free market principle, students are less able to experience classroom debate in a way that positively impacts their self-efficacy. Thus, we suggest that instructors consider both principles in coordination when planning classroom activities.

Similarly, the coding from students' experience from the fall semester illustrated multiple cases in which the Gestalt and uncertainty principles fostered opportunities for students gaining self-efficacy via enactive experiences. Because of these connections, we recommend instructors not only employ these principles, but consider how they might impact students when planning and presenting the problems in class. The instructor needs to be aware of, and attend to, students' affect when employing the Gestalt and uncertainty principles (Sriraman, 2005), attending to the potential frustrations of developing mathematical agency (Boaler, 2002).

The type of pedagogy should be taken into consideration in how the five principles are enacted. For example, the two courses investigated were both inquiry-based, and that may have been why many of the instances of the scholarly and free-market principles occurred. However, there is a case in which an instructor primarily used lecture-based pedagogical techniques coupled with a saturated emphasis on mathematical creativity (Omar et al., in press). He gave reoccurring assignments with open-ended questions and asked the students to write their reflections on their problem-solving process. He would give more grade-weight on the reflective piece, and students transitioned into feeling more like mathematicians which may be related to building self-efficacy.

Although we found evidence of effective use of each principle, we did not have a way to more precisely determine the impact of the principles on self-efficacy in the classroom. We observed cases in which various non-classroom factors appeared to determine how students were influenced by the principles. According to Sriraman (2005), these principles "can be applied in the everyday classroom setting" (p. 26). Therefore, we are considering broadening the "definitions" of these principles to include influences outside the classroom as well. Additionally, evidence of the association between the five principles and vicarious role-modeling toward others outside the classroom highlights the importance of these principles in changing students' attitudes toward mathematics. Fannie and Francisca actively engaged their peers in a way that demonstrated their care for the subject, potentially changing both their own, and others', perceptions of mathematics.

Finally, we would like to investigate the quality of implementing such principles. For example, one could look at all five principles and implement them like a checklist. The instructor could then claim that they implemented the principles and valued creativity. However, the consistency and affectual support that was observed in these classes demonstrated a different quality of implementation. Thus, we are interested in future research developing more refined ways to measure the quality or impact of the five principles both in and out of the classroom.

5. Conclusion

In this paper, we presented evidence qualitative connections between the five principles for fostering mathematical creativity and described a potentially robust way to quantitatively study these connections. Because of the limitations in the amount of quantitative data gathered thus far, we aim to continue this research on a larger scale. By providing a better understanding of the strength of the correlation between the five principles and increased self-efficacy for proving, such research can guide instructors and administrators in designing and implementing pedagogy. Valuing mathematical creativity in the classroom appears to offer better support for students' mathematical development and identity, which may have large impacts to mathematics and other STEM fields.

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