HOW CAN WE ASSESS UNDERGRADUATE STUDENTS' CREATIVITY IN PROOF AND PROVING?

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Students' creativity in mathematics is well considered in the K-12 level, yet there is a difference between K-12 creativity and creativity employed by mathematicians, which undergraduate students in proof-based courses may experience (Sriraman, 2005). *Our research group has created a creativity-in-proving rubric (CPR) to frame creativity within proof and the proving process. Finally, we reveal some results from our ongoing study on fostering creativity in a transition-to-proofs or introductory undergraduate proofs course* (Moore, 1994). *We believe this rubric might be helpful, not only to undergraduate courses, but perhaps secondary school proof courses or K-12 classes that emphasize reasoning.*

Key Words: creativity proof rubric, undergraduate mathematical creativity, creativity in proving

BACKGROUND LITERATURE

Creativity is an important aspect of the growth of undergraduate students' learning of mathematics. When teaching, avoiding the acknowledgment of creativity could "drive the creatively talented underground or, worse yet, cause them to give up the study of mathematics altogether" (Mann, 2005, p. 239). Furthermore, not exposing students to creative proofs or solutions to problems could lead them to believe that the study of mathematics is about procedures and somehow remembering the correct proof technique. Although much has been researched about creativity in the K-12 literature (e.g., Silver, 1997; Lev-Zamir & Leikin, 2013), according to Sriraman (2005), there is a difference between K-12 creativity and mathematical creativity, i.e. the creativity skills mathematicians employ. We claim that proof-based courses aim to be somewhat closer to what a mathematician does than K-12 mathematics in the continuum of a mathematician's career.

We aim to add to the undergraduate mathematics education literature about creativity, particularly in the context of proving, by creating a creativity-in-proof rubric (CPR). This rubric is a product of a larger study on teaching and learning creativity in an introduction to proving course (these courses are described in detail by Moore (1994)).

CPR ORIGINS

As a research group, we were intrigued by a creative thinking rubric created by the American Association of Colleges and Universities (AAC&U) (Rhodes, 2010). This rubric was created to record growth and value creativity in essays written in a student's first and fourth years of his/her university studies. The AAC&U creativity rubric focuses on six different areas (acquiring competencies, taking risks, solving problems, embracing contradictions, innovative thinking, connecting-synthesizing-transforming). Two researchers on our team interviewed active research mathematicians about creativity in proof, and in particular, allowed them to examine the aforementioned rubric to value creativity in three sample proofs. We utilized these experts' ideas to eliminate some categories and revise others to make it specific to mathematics content.

With regards to the influential literature on creativity in mathematics, we examined another rubric (Leikin, 2009), which describes three aspects of mathematical creativity: fluency (how well-prepared you are for the problem), flexibility (how many correct or incorrect solutions created for the problem), and originality (how novel your solution is). Finally, we included aspects of the Torrance tests on creativity (Torrance, 1966) in our definitions of the created categories. We leveraged these aspects to the revised rubric from interviews and created our first version of the CPR.

CPR EXPLANATION

Our research group is taking the definition of creativity to be that of Sriraman (2005): "(a) the process that results in unusual (novel) and/or insightful solution(s) to a given problem or analogous problems, and/or (b) the formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle requiring imagination" (p. 24). Our CPR has three categories, each with three hierarchical levels of evidence of creativity. These categories and levels were the result of first coding one student's proving attempts of four theorems, then coding four different students' proving attempts of a theorem from an introduction to proving course taught by one of the authors. The proving attempts were obtained using LiveScribe pens, which capture synchronized writing and audio. The pens allow the researchers to see when and on what part of which page a student works on, hence we can examine students' proving processes instead of the final written proof (Savic, 2012).

The first category is *Taking Risks*, which we define as *the ability to approach a proof and demonstrating flexibility in using different approaches*. Students in the beginner level exhibit <u>at least one</u> of these proof properties or proving actions: make an attempt at a proof (e.g., listing givens, providing few numerical examples), provide only one incomplete proof (with some left-out chain of reasoning), or make an attempt to use one proving technique (e.g., contradiction, induction, cases). Students in the developing level exhibit <u>at least one</u> of these properties: state hypothesis and conclusion of the theorem while providing applicable theorems/definitions/examples, provide a complete argument without a rigorously written proof, or implements at least one proving technique completely. Finally, students in the satisfactory level satisfy the criteria in the developing level and also exhibit <u>at least two</u> of these properties: attempts to develop a new object for the proof, attempts a "trick" learned or used in a previous proof, indicates multiple approaches to proving, provides a full subproof, or evidence of going back and forth between the hypothesis and the conclusion of the theorem.

We define the second category, *Making Connections,* as *the ability to demonstrate links between multiple representations, ideas from the current course that the student is in, and possible prior knowledge from previous courses*. Students in the beginner level exhibit <u>at least one</u> of these properties: make an attempt to recall related definitions/theorems/graphical representations, indicate that they recall some similar idea from a previous course, or attempt to make connections between two different representations of a concept in order to develop a proof. Students in the developing level exhibit <u>at least two</u> of these properties: recognize relevant definitions/theorems that help in the reasoning, making connections between relevant definitions/theorems, demonstrate various connections between representations of concepts, make connections between specific examples and general cases. Students in the satisfactory level satisfy the criteria in the developing level and also exhibit at least two of these properties: implement appropriate definitions/theorems correctly, combining different proving techniques (e.g., contradiction, induction, cases), demonstrate an abstract connection between proven ideas or proof techniques, use different representations in the final proof, or implement another idea from another course or topic.

The final category, *Posing Questions/Conjectures*, is defined as *the ability to state a mathematical question that is either pertinent to the proof or can be proven itself*. Students in the beginner level exhibit at least one of these properties: pose a conjecture that demonstrates an incorrect use of definition, reword a theorem or posing clarifying questions. Students in the developing level exhibit at least one of these properties: pose conjectures in relation to previous theorems in the form of corollaries or pose a proving process question. Students in the satisfactory level exhibit at least one of these properties: pose a conjecture that leads to a generalization, pose an original/novel idea, or pose a question that calls for investigation.

We now show an example of using the CPR on a student's proving process in the introduction to proving course.

EXAMPLE OF CPR USE

We discuss some of the proving actions that Student 10 enacted chronologically for Theorem 29 (Thm 29), "If 3 divides the sum of the digits of n, then 3 divides n." In fewer than 30 minutes, Student 10 first attempted directly proving the statement by creating a representation for n: $n = a_m \cdot 10^m + \cdots + a_1 \cdot 10^1 + a_0$ and then moved on to prove the next theorem in the assignment. After coming back, he enacted these actions in his second attempt of proving Thm 29 (note the italics are time-stamps of the actions, with minutes first and seconds after):

Figure 1 – Student 10 Written Work. Line numbers are added for reference.

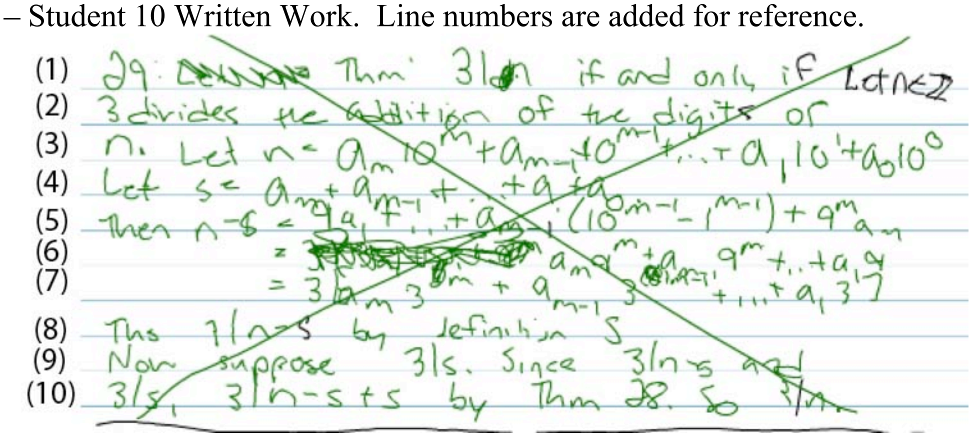

Table 1 – Student 10 Proving Actions

Lines (1) – (3):	"Let n," then scratches that out, then takes a short pause. "$3\|n$ if and only if 3 divides the addition of digits of n. Let $n = a_m 10^m + a_{m-1}10^{m-1} + \cdots + a_1 10^1 + \ a_0 10^0$
Line (4):	Let $s = a_m + a_{m-1} + \cdots + a_1 + a_0$" *(End 2:59)*
Line (5):	Continues next day with "$n - s = 9a_1 + \cdots + a_{m-1}10^{m-1} +$

	$9^m a_m$"
Line (6):	Writes something unclear…then scratches it out. Then writes "$= a_m 9^m + a_{m-1}9^{m-1} \ldots + a_1 9$" *(Pause 3:14-5:55)* This is followed by scratch work on powers of 3 and 9 with short pauses in between until 8:40
Line (7):	He writes "$= 3\|$" *(Pause 8:40-8:55)* Then back to his scratch work: $3 \cdot 3^4 = 3^5$ *(pause)* $9^2 = (3^2)^2 = 3^4$ *(pause)* $9^3 = (3^2)^3 = 3^6$
Line (7):	He writes in the proof: "$= 3\|a_m$" *(10:26)* Short pause and goes to scratch again and writes 9^5
Line (7):	Goes back to append 3^{2m} "$=3\|a_m 3^{2m}$" *(10:52) (Pause 10:56-12:31)* He verbally stated: 'How do I know..see this..I feel so stupid' *(11:40)* 'I can't believe I can't factor a 3 out of this…' then erases powers of 9 in his scratch work *(12:03)*
Line (7):	Finish writing "$3\|a_m 3^{2m} + a_{m-1}3^{(2)m-1} + \cdots + a_1 3^1$ *(12:43)*
Line (8):	Thus $9\|n - s$ by DEF S *(13:22-15:02)*
Lines (9) and (10):	Now suppose $3\|s$. Since $3\|n - s$ and $3\|s$, $3\|n - s + s$ by Thm 28. So $3\|n$" *(15:46)* He goes back to erase the squares from $3^{2\mathrm{m}}$ and $3^{(2)\mathrm{m}-1}$ *(16:20)* He says: 'That was hard,' *(16:41, finishes at 16:50)*

The student again revisits the proof a third time later that night, which we did not provide in this paper. For brevity, we focus on the second attempt of proving Thm 29 by Student 10. It is important to highlight that the rubric is created and utilized to investigate and assess the complete proving process rather than a student's final proof.

We coded this student's proving process as satisfactory (Level 3) under the *Taking Risks* category because, not only did he satisfy the developing category by providing applicable theorems and calculations, he demonstrated multiple attempts in proving Thm 29 and attempted to create or algebraically manipulate items for proving success (e.g., labeling s, 3^{2m} instead of 9^m). We believe the student's proving process is developing (Level 2) under *Making Connections* due to recognizing relevant definitions (DEF S) and theorems (Thm 28) and demonstrating connections between these definitions and theorems. We believe Student 10's process did not provide enough evidence to be satisfactory in this particular category (recall that to be in satisfactory level for *Making Connections,* a process needs at least two aforementioned properties). Finally, we coded this student's proving process as developing (Level 1) on *Posing Conjectures/Questions* due to the conjecture written that "$3|n$ if and only if 3 divides addition of digits of n," which is a minor expansion of what was written in the theorem, "If 3 divides the sum of the digits of n, then 3 divides n."

POTENTIAL USES OF CPR IN THE MATHEMATICS CLASSROOM

The CPR is designed to assess students' abilities to take mathematical risks; make connections between different courses, topics, problems, solutions, or methods; and problem posing or conjecturing abilities. We envision using the CPR as a formative assessment tool to promote and enhance students' creativity in proving. As a formative assessment tool it would also provide instructors feedback on how tasks and curriculum practices promote their students' creativity. However, for the CPR to be an effective tool, the learning environment in the classroom must be one that fosters such creativity. Setting up this environment includes establishing socio-mathematical norms (Fukawa-Connelly, 2012). A socio-mathematical norm we implemented in the introduction to proving course was to ask students to make conjectures based on certain mathematical concepts. For example, after covering some elementary properties and theorems of sets, we presented the students with the task, "If you had four or more sets, give me one theorem you could pose and prove that theorem." Another socio-mathematical norm was to allow students around 20-25 minutes of uninhibited and non-judgmental "playtime," which was time allotted to think about a problem posed in class and to write down everything that they thought or knew would be helpful. Both norms are established and then enacted by many students outside of class. Additionally, in order to effectively implement the CPR, the instructor must determine if the task allows for creativity. That is, the task must be properly chosen so that students have the opportunity to display their creative thinking skills. For instance, there are multiple methods (induction, contradiction, direct proof) to prove Thm 29. Finally, we believe that the CPR should be shared in the classroom and perhaps given some percentage of the total grade. If a teacher explicitly values mathematical creativity, the students may also, which can lead to their improvement in mathematical proving.

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