The Creativity-in-Progress Rubric (CPR) on Proving: Two teaching implementations and students' reported usage

Abstract:

A growing body of mathematics education research points the importance of fostering students' mathematical creativity in undergraduate mathematics courses. However, there are not many research-based instructional practices that aim to accomplish this task. Our research group has been working to address this issue and created a formative assessment tool, the *Creativity-in-Progress Rubric (CPR) on Proving*. This tool is developed to help students evaluate their own proving process as well as provide instructors insight into students' self-evaluation and process. In this paper, we provide a brief description of the CPR on Proving and explain its implementation in two courses at different institutions. Additionally, we share several students' usages of the CPR and provide some practical implementation suggestions.

Keywords: Formative assessment, mathematical creativity, proof, undergraduate mathematics education

1 INTRODUCTION

The goal of this paper is to share the implementations of the Creativity-in-Progress Rubric (CPR) [5] by two instructors from our research group at different institutions and provide practical suggestions for implementations. These implementations are provided to show examples of how the CPR can be used by instructors who consider incorporating mathematical creativity language during discussions in their proof-based courses. In addition to implementing the CPR in two classes, we conducted individual interviews with five students who took these courses. We include some students' comments about their usage of the rubric in these courses. We claim that both these implementations and the feedback generated by the interviews may assist a practitioner who is considering explicitly discussing creativity in his/her proving course. Before proceeding any further, we would like to provide some motivation for using the CPR.

The Mathematical Association of America's Committee on the Undergraduate Program in Mathematics has emphasized the importance of mathematical creativity in its latest guidelines [16]. The guidelines state that "[a] successful major offers a program of courses to gradually and intentionally leads [sic] students from basic to advanced levels of critical and analytical thinking, while encouraging creativity and excitement about mathematics" (p. 9). Under *Cognitive Goals and Recommendations*, the guidelines also state "these major programs should include activities designed to promote students' progress in learning to approach mathematical problems with curiosity and creativity and persist in the face of difficulties" (p. 10).

Nadjafikhaha et al. [10] provide an even stronger claim: one of the goals of any educational system should be to foster mathematical creativity. However, as much as mathematical creativity is discussed as an important aspect in undergraduate mathematics (e.g., [19]), its explicit implementation in classrooms is rarely mentioned or studied. In his book chapter, Mathematical Creativity [3, p. 53], Ervynck stated, "[W]e therefore see mathematical creativity, so totally neglected in current undergraduate mathematics courses, as a worthy focus of more attention in the teaching of advanced mathematics in the future." Students also expressed this sentiment. For example, in interviews from one of our previous studies a student said:

> This [inquiry based learning class] is definitely more conducive to creativity than a traditional course structure . . . as far as learning things goes, there is not much creativity in the things that I've learned previously. They have ways to do things and while I can learn them and learn how they work, I have not used my own creativity to find them.

This feedback from the student identifies the potential of Inquiry Based Learning (IBL) courses to foster mathematical creativity. In these courses, classroom discussions make an integral part because they may "increase student learning, motivate students, support teachers in understanding and assessing student thinking, shift the mathematical authority from teacher (or textbook) to community" [1]. These discussions also give the instructor an opportunity to promote aspects of creativity discussed in research [4, 5, 10, 12, 13, 15, 17, 19]. However, there is still a need to understand how such discussions could be structured to develop and enhance students' mathematical creativity. To facilitate such discussions, as well as help students to push their own creativity, our research group developed a formative assessment rubric, the CPR on Proving, which went under several revisions [5, 13, 14, 15].

In following sections, we describe the CPR on Proving (for a more detailed report of its development, see [13]), and then share its implementation in two classes. We provide a discussion on the class environment that was crucial to the implementation of the rubric.

2 CPR ON PROVING

Though the rubric was initially created to be used by researchers and instructors, it has been imported into the classroom since we believe it can serve as a tool to open discussions with students about the development of their own mathematical creativity. It can also show students the practices recommended by mathematicians and creativity literature [14, 17] that may enhance their mathematical creativity. Thus, the name "rubric" in the title indicates a formative assessment for the student, rather than a judgement on his/her creativity.

From our previous research, we found two processes, **Making Connections** and **Taking Risks**, which can contribute to the development of mathematical creativity in proving [14, 15]. Making Connections is defined as *the ability to connect the proving task with definitions, theorems, multiple representations, or examples from the current course that a student is in or possible prior experiences from previous courses.* Taking Risks is defined as *the ability to actively attempt a proof, demonstrate flexibility in using multiple approaches or techniques, posing questions about reasoning within the attempts, and evaluating those attempts.* These two categories have subcategories that are reflective of the different aspects of creativity found in prior research. In particular, they are designed to have students explicitly think about ways to develop aspects of mathematical creativity such as *flexibility, fluency*, and *originality* [18].

Tables 1 and 2 show the two categories, Making Connections and Taking Risks, with their subcategories. For each subcategory, the rubric provides three general levels: *Beginning, Developing*, and *Advancing*, each serves as a marker along the continuum of a student's progress in that subcategory. This continuum among levels of the rubric communicates the possible states of growth and it allows for a better approximation of placing proving attempts on the rubric based on the work provided. The user of the rubric can indicate the corresponding level by tracing the arrow using a highlighter or a marker.

	Beginning	**Developing**	**Advancing**
Between Definitions/ Theorems	Recognizes some relevant definitions/theorems from the course or textbook with no attempts to connect them in their proving	Recognizes some relevant definitions/theorems from the course and attempts to connect them in their proving	Implements relevant definitions/theorems from the course and/or other resources outside the course in their proving
Between Representations [1]	Provides a representation with no attempts to connect it to another representation	Provides multiple representations and recognizes connections between representations	Provides multiple representations and uses connections between different representations
Between Examples	Generates one or two specific examples with no attempt to connect them	Generates one or two specific examples and recognizes a connection between them	Generates several specific examples and uses the key idea synthesized from their generation

Table 1. Creativity-in-Progress Rubric on Proving: Making Connections

[1] We define a *mathematical representation* similar to NCTM's (2000) definition. It includes written work in the form of diagrams, graphical displays, and symbolic expressions. We also include linguistic expressions as a form of lexical or oral representation. For example, a student can use the lexical or oral representation, "the intersection of sets A and B"; a Venn Diagram to depict his/her mathematical thinking; a symbolic representation $A \cap B$; or set notation $\{x|x \in A \text{ and } x \in B\}$ (which is also a symbolic representation). Note the last two representations are in the same category, e.g. symbolic, but they are still considered two different representations.

	Beginning	Developing	Advancing
Tools and Tricks[2]	Uses a tool or trick that is algorithmic or conventional for the course or the student	Uses a tool or trick that is model-based or partly unconventional[3] for the course or the student	Creates a tool or trick that is unconventional for the course or the student
Flexibility[4]	Begins a proof attempt (or more than one proof attempt), but uses only one approach	Acknowledges and/or uses more than one proving approach, but only draws on one proof technique	Uses more than one proof technique
Posing Questions	Recognizes there should be a question asked, but does not pose a question[5]	Poses questions clarifying a statement of a definition or theorem	Poses questions about reasoning within a proof
Evaluation of Proof Attempt	Examines surface-level[6] features of a proof attempt	Examines an entire proof attempt for logical or structural flow	Examines and *revises* an entire proof attempt for logical or structural flow

Table 2. Creativity-in-Progress Rubric on Proving: Taking Risks

[2] Based on the Originality category from [9].

[3] Learned in a different context.

[4] A proof attempt is a continuous, sustained line of reasoning focused on a single theorem or conjecture. A proof approach is a proof attempt in which a new or different (to the prover) idea is introduced. Finally, a proof technique is a proof approach that addresses the overall logical structure of the proof. Common proof techniques include induction, proof by cases, direct proof, contradiction, and contrapositive.

[5] For example, a student writes a “?” next to something.

[6] Surface-level features include technical, computational, and line-to-line logical details.

3 IMPLEMENTATION OF THE CPR

As discussed earlier, two instructors implemented the CPR on Proving in two different classes at two different institutions. We should emphasize that we did not conduct these implementations as a comparison study as we had no intention to compare implementation methods and related student feedback. Rather, we provide two examples of how it can be used and how instructors may use it as they see fit in their curriculum and teaching styles. In the following, we provide a description of the implementation by each instructor.

3.1 Implementation I

In this implementation, the rubric was promoted as a tool for students to discuss their own work or other students' work. The instructor used the rubric to explicitly value mathematical creativity, and to have students see themselves as creative doers of mathematics. This section will discuss classroom transcripts as examples for future implementers on how the rubric's language can be explained and how the CPR can be used to discuss others' work; the next implementation will discuss how students used it on their own work.

One member in our research group implemented the CPR on Proving in a transition-to-proof course at a small liberal arts college in Spring 2015 with 19 students (12 males, 7 females). It was taught using Inquiry-Based Learning (IBL) where "students in class spent time giving and listening to student presentations, working in small groups, discussing ideas that generally arose from a group problem or student-presented solution" ([8, p. iii]). To help collect data, every student used a LiveScribe© pen for her/his homework, notes, and exams.

Prior to this course, students generally experience mathematical courses that are calculations-focused, rather than proofs-focused, and usually through lecture. As such, the instructor chose to spend the first part of the semester supporting the development of an environment in which students might become comfortable with the idea of proofs and also the IBL teaching pedagogy. Thus, students received a copy of this version of the rubric[1] during week 7 of the 15-week semester. However, before the rubric was distributed to the class, the instructor used phrases from the rubric with the intention of easing her students into the vocabulary used in the CPR on Proving. The idea was that by the time the students received the rubric, they would be somewhat familiar with the language. For example, if a student made a connection to a previous theorem, the instructor might say, “the student implemented theorem X in his/her proof.”

To help students familiarize themselves with the rubric, the instructor created scratch-work for a hypothetical student’s proof attempt and asked students to use the rubric to evaluate the student work during an exam in week 8. This particular question on the exam was not graded for correctness and all students who completed the question received a full score.

In Appendix A, there are several classroom transcripts provided to show examples of how the rubric was incorporated in the classroom. Transcript 1 is provided to show how the instructor clarified what constituted a “key idea synthesized from example generation” in subcategory *Between Examples* of the Making Connections category of the CPR. In this vignette, the class chose to discuss Student 4’s work on the theorem, “Let $n \in \mathbb{Z}^+$. If 3 divides n, then n is a trapezoidal number.” (See Figure 1).

Transcript 2 shows discussion on *Flexibility*. The instructor asked for justifications from students about what level they ranked the sam-

[1]For the final version of the rubric see [5]

ple proof in this subcategory. In this case, the answers from students changed the instructor's own ranking from Beginning to Developing.

In Transcript 3, the instructor addressed the fact that one part of the proof can be marked in more than one subcategory; in this case it was *Tools and Tricks* and *Between Examples*. Furthermore, not every proof attempt can be marked in all the subcategories. For example in Transcript 4, the proof attempt in Figure 1 did not provide any evidence for marking a level on the *Posing Questions* section.

The discussion of Student S4's work continues in Transcript 5. Questions about *Between Representations* came up and Transcript 5 shows how the students were able to help each other understand the meaning of the subcategory. From this classroom discussion, we see the students using the rubric to evaluate student work and to justify their ratings.

It did not matter much what the actual ratings were; the main purpose was that the students saw other students' work and that the CPR on Proving provided a way for them to discuss these proof attempts, while also giving the instructor a chance to promote aspects of creativity found in the literature [4, 5, 10, 12, 13, 15, 17, 19]. For example, Transcript 1 shows the instructor taking time to value creating examples and generalizing them to create a key idea that was used in the proof. By explicitly valuing this practice, the students can see the importance in making connections between examples. The rubric serves as an important tool to bring these practices to the attention of the students.

3.2 Implementation II

Another member of our research group implemented the CPR on Proving in Fall 2015. It was used in a proof-based course on Elementary Number Theory in a comprehensive private institution that has emphasis on liberal arts, applied sciences, and engineering. The class had 6 students (3 males, 3 females), all majoring in Mathematics. It was taught using IBL through the MAA text, Number Theory Through Inquiry. The students in this course had taken a transition-to-proof class before taking this course. Unlike at the other institution, the rubric was given to students as a "rubric on proving" and the word "creativity" was removed from the title. The reason behind this was to see if students would relate the rubric to mathematical creativity in one-on-one interviews, which only one student volunteered for.

A typical class was structured as follows: the instructor assigned a total of about 5 exercises or theorems for each class, which met twice a week for 75 minutes. Students' attempts were graded on completion for their homework grade. The rubric was introduced to students in the third week of the semester. The instructor explained what each subcategory meant in one class period while giving hypothetical examples. If a student had a complete (successful or unsuccessful) attempt of the proof on the homework, the instructor would ask him/her to present it on the board. Then the instructor would ask the students to discuss the proof by pointing out which subcategories, from the CPR on Proving, they noticed in this proof. The instructor sometimes led the discussion by asking more in-depth questions such as: "Can you point out a connection to a specific definition/theorem/example," or "can you describe a tool or trick that was used in this proof?", etc.

If no student completed his/her proof attempt, the students would work on the proof in class with guided intervention from the instructor. To accelerate the proving process, the instructor would ask students some of the following questions: "Can you make a connection to a previous definition/theorem," "Can you use examples to understand the statement of the theorem or to figure out the key idea(s)," or "Can you try to use another proof technique?"

The students were formally asked to use the rubric on five occasions throughout the semester. For example, they used the rubric on their own proving attempts and submitted their evaluations as part of their homework. In another instance, the instructor demonstrated two proof attempts of the following theorem: if $2^n + 1$ is prime then n must be a power of 2. In the third class period, the instructor showed students a complete and correct proof done by the instructor. Students were asked to use the rubric on this proving task. As the instructor switched his proof approaches and tried to make a connection to a previous proof that uses an algebraic trick of factoring $x^n - 1$, the students' usage of the rubric on this task opened a discussion on many of the subcategories in the rubric, specifically the subcategories Flexibility and Tools and Tricks. The discussion concluded by emphasizing the importance of how utilizing the rubric can be helpful when getting "stuck" during a proving process.

As in the first implementation, every student used a LiveScribe© pen for her/his homework, notes, and exams. In Figure 2, student T5's work is demonstrated for the Theorem: For integers a, b, n, and x, $ax \equiv b \pmod{n}$ if and only if $gcd(a, n)|b$. We use this example to show how the rubric can be used by students in evaluating their own work. The student is examining his attempts on both the surface-level features and the logical flow of their proofs. He is evaluating his proof attempt when he writes "This is correct" and "b doesn't have to be the gcd mistake." The student is also checking the surface-level features as he moves along the proving process as we can see from the scratched out work.

On the final exam, the instructor asked the students to again prove the theorem "If 3 divides the sum of the digits of n, then 3 divides n" and then explain briefly which subcategories of the CPR on Proving they used in their proofs. The students had worked on the proof of this theorem in Chapter 1 (Division and Euclidean Algorithm) and in Chapter 3 (Modular Arithmetic). The rubric was attached to the final and the students had access to the textbook during the exam but did not have access to their notes, which included their previous proofs. Student T4's work can be found in Figures 3 and 4.

The student wrote down the question "How do you write the sum of digits without base 10 then translate them to base 10?" to reason through her proof. She also worked out some examples to understand the statement. We believe that the CPR on Proving was the trigger that reminded the student to take both of these actions because this relates to the making connections category on the rubric. These two samples explicitly show how the CPR on Proving can be used by the students as a "checklist" to overcome difficulties through the proving process.

4 RECOMMENDATIONS FOR CLASSROOM USE

Just as students can use the CPR on Proving as an assessment tool, instructors can as well. The CPR on Proving can be utilized as a formative assessment tool to encourage students to engage in the practices both mathematicians and the creativity literature discuss that may foster the development of mathematical creativity. For instance, as demonstrated in the discussion above, while students presented their attempts or evaluated others' work, the instructor and students pointed out the places where they made (or did not make) a connection or took (or did not take) a risk. Using the discussion about the "key idea synthesized from generating the examples" that occurred at Institution I, the instructor used this opportunity to explicitly value the practices that mathematicians believe lead to creativity [17].

It is important that instructors communicate the idea that the CPR on Proving does not measure a student's overall creativity or evaluate the validity of a final proof. Rather, the emphasis is on the proving process and the development of practices that mathematicians report can lead to creative thinking.

The CPR on Proving can be used with any proof task; however, the CPR on Proving may yield more information for students and instructors when used on tasks that elicit exploration or when the key idea(s) of the proof are not directly seen. When such tasks are assigned for an in-class activity, students can use the CPR on Proving to reflect on their or their peers' proof production by making the categories the basis for discussion. We recommend that instructors pose theorems that have multiple proofs or tasks with multiple solutions. For example, the theorem: if n is an integer, then $n^2 - n$ is even. They can prove it directly using cases or re-writing it so that $n(n-1)$ is a product of two consecutive integers. If given freedom and proper support, at least one student will come up with another method of proving.

Instructors may choose to emphasize and focus on specific subcategories and not necessarily the entire rubric. For instance, at Institution I, the instructor chose one subcategory in the rubric, Making Connections to Definitions and Theorems, to focus on. The instructor showed students' work that made a connection to another theorem as opposed to using the definition to prove it and asked students to comment on it. By highlighting that additional method and asking students to reflect on the connection made to the previously proved theorems, other students may start attempting this in their future proofs. It is the role of the instructor to expose and point out different ways of thinking in the classroom and the CPR on Proving is a useful tool for this purpose. Moreover, at Institution II, some subcategories were also highlighted more than others throughout the semester. Making Connections with

Definitions, Theorems, or Examples; Tools and Tricks; and Evaluation of the Proof Attempt were discussed and used the most. The majority of the proving tasks throughout the semester did not require different representations or different proof approaches and hence two of the subcategories were not emphasized as much. The subcategory Flexibility was only emphasized by the instructor when students were "stuck" using a proof technique that did not help in the proving process and they needed a reminder to change their proof technique.

The instructors in both implementations emphasized the proving process and not just the final proof. They explained to students the CPR on Proving was not created to assess "correctness" or "validity." This is because mathematical creativity may not lie in the student's final proof, but rather in the winding path that the student's work takes. Many students can be incredibly creative and create invalid proofs. Ervynck [3, p. 53] stated "[a] major characteristic of mathematical creativity which distinguishes it from the generally accepted qualities of a mathematical theory is that it is sometimes fallible." Hence, it is important to emphasize and remind students, via the rubric, that mistakes or incorrect attempts can lead to successful and creative proofs.

We want to endorse the idea that in the natural process of producing a conjecture or proof, students may produce wrong attempts. We believe that it is important for instructors to have grading schemes that encourage students to keep persisting, even after producing a wrong attempt. Therefore, we encourage a grading scheme that is asset-based rather than deducting points, because it may discourage trying different attempts. One way of doing that is by allowing students to redo work. For example, a homework problem could be worth a total of 2 points: attempts are worth 1 and a correct answer is worth 1. Students may work on the proof several times until they produce a correct answer. This is in contrast to the traditional grading systems which emphasize “taking away points” from students. This approach may seem punitive and may discourage students from trying new approaches, or diverging from “the method that is taught by the professor”. In a sense, by explicitly highlighting taking risks and having a grading system that encourages this behavior, we give the students “permission” to try a new technique without negative effects on their grades.

In the next section, we show how students in our pilot studies reported that they used the rubric as a “checklist,” often referring back to it when they felt ”stuck.” Impasses are an inevitable aspect of proving, and having a tool to assist or guide someone through the struggle may be beneficial.

5 STUDENTS’ INTERPRETATIONS OF THE CPR ON PROVING

Four students (S1, S2, S3, S4) from Institution I and one student (T1) from institution II volunteered to be interviewed via Skype after they completed the courses. Both sets of interviews were conducted by a third researcher in the group who was not the instructor of either implementation. In the following, we provide students’ feedback and how they perceived the rubric. (Emphasis in bold is added by the authors).

When asked about what it means to be creative in mathematics, Student S4 linked trying different things to building creativity:

> For example, if you have a proof, and you try a direct proof, well **try something different!** Do the contrapositive, or do the contradiction. You know, even if it may not work and in the end you spent an extra 20 or 30 minutes to do it, you know, it pays off in the end and it **builds your creativity**.

S4 then added:

> [W]ell, I would kind of use it [CPR] as a **checklist** to go through it and when I'm **evaluating my proof**, I would use and say "could I **make any connection?**" … You know, but could I do more? Could I do it better? Could I go from **developing to satisfactory** in my proof?
>
> When I got stuck on the proof on a problem in the book, I would just look back to this [CPR], and 'oh let me try it this way, let me try it that way.'

Student S3 talked about how the rubric made him aware that "doing" different things was encouraged:

> It [the rubric] lets me know that, you know, it's okay to go between examples, it's **ok to do this, it's ok to do that**.

In addition, Student S1 talked about notions of Flexibility:

> I mean, **thinking about [a theorem] in different ways** and proving it in different ways is the whole point of being a mathematician, is being able to prove something.

When asked to discuss the rubric, Student T1 said:

> They [printed copies of rubric] were a **good way to assess the work I was doing**, to be able to see exactly the issues that I was struggling with …

The aspects of the rubric that student T1 found to be the most useful was:

> … I would get disheartened after one or two failed attempts, … that would cause me to stop. … It just seemed that after spending 20 or 30 minutes on a problem, it was no longer beneficial

> for me to continue. So it was definitely an insight into myself, I guess, **using the rubric**.
>
> ... **relating back to theorems and mathematical tricks** because, that would make the problems easier

Since students were not given the rubric as a rubric on mathematical creativity at Institution II, student T1 was asked if he saw a relation between the rubric and mathematical creativity. His answer was affirmative:

> It was definitely the "tips" and **tricks**, whatever the first one is. Being able to **relate to previous theorems**, and using little tricks like, ...

6 DISCUSSION

The interviews indicated some of the benefits and areas of improvement of our implementations. Students used the rubric as a prompt to think about their proving tasks in different ways, and they considered it as a checklist to remind themselves of the actions they could take when they got stuck. In particular, through the CPR on Proving, students can reflect and improve their proving processes. The rubric gave students a sense of agency; students like S3 were more apt to contemplate taking these actions knowing that they were allowed to in the classroom. If not agency, students like S4 also thought of the rubric for personal achievement in their proving processes. In either case, the rubric provided reminders of possible courses of action for the students. Those considerations can play a big role in their future proof-based courses.

The instructors explained to students that the CPR on Proving was not created to assess correctness. Students (see S3's quote) were aware that mistakes can lead to successful and creative proofs and they were encouraged, through using the rubric, to try different things. Moreover, by explicitly having a grading system as explained in Section 4, students (See S3's quote) were given "permission" to try a new technique without negative effects on their grades.

On the other hand, the instructor of the second implementation observed that it was challenging to convince some students to try another proof technique/approach if they were successful in the first time. This happened because the instructor did not relate (on purpose) the rubric to mathematical creativity. Student T1 stated:

> One of the things was that the student shows that they are continuing to work through a theorem in multiple ways, try different types of work. If I get a theorem in my first attempt in 5 minutes, based on one of the easier methods, there's no real incentive for me to try one of the hard methods and spend more time on a theorem I already know the answer to. That stood out a little.

However, as S4 picked up in her quote above, even if a proof has been produced, can it be improved to produce a "better" one? This realization by the student is an additional benefit of using the rubric. This mathematical practice of producing a proof to an already proved theorem is consistent with those of mathematicians' [2, 11]. Student S2 says this can be a time consuming process:

> The [class] **pace** is kind of **slow**, for me. It's because, to evaluate one proof and then think, like **how can we improve this proof**, then that one half hour just passed, maybe we just finish two proofs or one proof.

This quote shows the rubric can be used to encourage students who finish their attempts on the first attempt to try and improve their attempt by either looking at it using a different proof technique or approach. It also highlights that creating proofs is a slow process. This is in stark contrast to many lectures where the solutions are presented by instructors, detached from the process of creating them. This encouragement and exposure to taking time to improve proofs will hopefully have a positive impact on the development of students' mathematical creativity.

In addition, some students reported having some difficulties using certain (sub)categories from the rubric. For example, student S3 stated "sometimes I want to use this [rubric] and I ask the professor: What does this category mean?" Student T1 from the second implementation also indicated the same issue: "I never really knew what to give myself for those [subcategories]... if the professor did some of the rubric with our proofs ... I feel like it would have helped with the way the assignments were graded, our understanding of that process, and of what they were looking for. ... Being able to get a whole understanding of the rubric, what each section is, as well as exactly what constitutes a lower end of the rubric." The instructors believe that if the rubric was introduced earlier and implemented more frequently, this issue might have been resolved. The same student (T1) had a logistical issue with the rubric: "I didn't like printing them out all the time." This problem has an easy fix: supply students with laminated copies of the rubric to be used with dry erase markers. This was considered at the first institution, but not in the second one.

In future papers, we are particularly interested in comparing the benefits of implementing the rubric versus the trade-offs, such as content coverage, of creating time for classroom discussions. However, some of the literature on IBL [7] indicate that "the benefits of active learning experiences may be lasting and significant for some student groups, with no harm done to others. Importantly, 'covering' less material in inquiry-based sections had no negative effect on students' later performance in the major" [7, p. 197].

7 CONCLUSION

Creativity in mathematics is important for both mathematicians doing mathematics and students' development of mathematical actions. Our rubric, the CPR on Proving, aims to allow students to self-create potential for mathematical creativity. Two instructors used the rubric differently in either class, but the goal was the same: a recognition or awareness of students' own proving processes. Instructor I established the vocabulary of the rubric prior to implementing it and using it as a discussion tool in class. Then students utilized the rubric to improve their work. Instructor II actively encouraged the students to use the rubric as a checklist, and students incorporated it into their assignments. Many students expressed positive notions of the rubric, perhaps due to the insistence of the rubric as a helpful tool and not as a "rubric for grading". We encourage the use of the rubric, if only to allow the students a respite from continued proving to reflect on their proving processes. We believe that the more a student is aware of his/her own mathematical thinking, the more mathematically creative that student may be.

REFERENCES

[1] Cirillo, M., 2013. What does research say the benefits of discussion in mathematics class are. *Research Brief* No, 19.

[2] Dawson, J. W., 2006. Why do Mathematicians Re-prove Theorems? *Philosophia Mathematica*, 14(3), pp.269-286.

[3] Ervynck, G. 1991. Mathematical creativity. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 42-53). Dordrecht: Kluwer.

[4] Karakok, G., Savic, M., Tang, G. & El Turkey, H., 2015. *Mathematicians' Views on Undergraduate Students' Creativity.* Proceedings of the 9th Congress of European Research In Mathematics Education. Prague, Czech Republic.

[5] Karakok, G., Savic, M., Tang, G., El Turkey, H., Plaxco, D & Naccarato, E., 2016. *A Rubric for Creativity in Writing Proofs.* Mathematical Association of America Focus Magazine, 36(1). pp. 42-43.

[6] Karakok, G., El Turkey, H., Savic, M., Tang, G., E. Naccarato, & D. Plaxco (in press). *Creativity-in-Progress Rubric on Proving - Enhancing Student Creativity.* 13th International Congress on Mathematical Education, Hamburg, Germany.

[7] Kogan, M. and Laursen, S., 2014. *Assessing long-term effects of inquiry-based learning: A case study from college mathematics.* Innovative higher education, 39(3). pp. 183-199

[8] Laursen, S., Hassi, M.L., Kogan, M., Hunter, A.B., Ethnography & Evaluation Research, & Weston, T., ATLAS Assessment and Research Center 2011. Evaluation of the IBL mathematics project: student and instructor outcomes of inquiry-based learning in college mathematics. *A report prepared for the Educational Advancement Foundation and the IBL Mathematics Centers.*

[9] Leikin, R., 2009. Exploring mathematical creativity using multiple solution tasks. *Creativity in mathematics and the education of gifted students*, pp.129-145.

[10] Nadjafikhaha, M., N. Yaftianb, and S. Bakhshalizadehc 2012. Mathematical creativity: some definitions and characteristics. *Procedia - Social and Behavioral Sciences.* Volume 31: 285-291.

[11] Rav, Y., 2007. A Critique of a Formalist-Mechanist Version of the Justification of Arguments in Mathematicians' Proof Practices. *Philosophia Mathematica*, 15(3), pp. 291-320.

[12] Rhodes, T.L., 2008. VALUE: Valid assessment of learning in undergraduate education. *New Directions for Institutional Research*, 2008(S1), pp.59-70.

[13] Savic, M., Karakok, G., Tang, G., El Turkey, H., & Naccarato, E., 2016. *Formative Assessment of Creativity in Undergraduate Mathematics: Using a Creativity-in- Progress Rubric (CPR) on Proving.* In R. Leikin & B. Sriraman (Eds.), Creativity and Giftedness: Interdisciplinary perspectives from mathematics and beyond, New York, NY: Springer.

[14] Savic, M., Karakok, G., Tang, G., Stubblefield, M., & El Turkey, H., 2014. *How Can We Assess Students' Creativity in Proof and Proving?*. Proceedings of the 8th Annual Conference of Mathematical Creativity and Giftedness. Denver, CO.

[15] Savic, M., Karakok, G., Tang, G., & El Turkey, H., 2015. *Developing a Creativity-in-Progress Rubric on Proving.* Proceedings of the 18th Annual Conference on Research in Undergraduate Mathematics Education. Pittsburgh, PA.

[16] Schumacher, C. and Siegel, M., 2015. *2015 CUPM Curriculum Guide to Majors in the Mathematical Sciences*, Mathematical Association of America.

[17] Tang, G., El Turkey, H., Savic, M., & Karakok, G., 2015. *Exploration of Undergraduate Students' and Mathematicians' Perspectives on Creativity.* Proceedings of the 18th Annual Conference on Research in Undergraduate Mathematics Education. Pittsburgh, PA.

[18] Torrance, E.P., 1998. *Torrance tests of creative thinking: Norms-technical manual: Figural (streamlined) forms A & B*. Scholastic Testing Service.

[19] Zazkis, R., and Holton, D., 2009. Snapshots of Creativity in Undergraduate Mathematics Education. In R. Leikin, A. Berman, & B. Koichu, *Creativity in mathematics and the education of gifted students*, pp. 345-365. Rotterdam, the Netherlands: Sense.

APPENDICES

APPENDIX A: Class Transcripts

Transcript 1.

Student 5 (S5): Student 4 was really cool too. I like the way [Student 4] separated the m's and then made it $m-1$, m, and $m+1$. I didn't even think of doing that.

Instructor (I): Yeah, and you know what? That came out of the examples.

S5: Yeah.

I: Look at those examples. What did you guys mark for Examples for Student 4?

S5: Advancing, I think.

I: Student 4, question 5?

Student 6 (S6): Advancing.

S5: Mm hmm.

I: Let me read "Advancing" to you. "Utilizes the key idea synthesized from generating the examples." You can see—do you see examples in the middle where it says scratchwork?

S5: Mm hmm.

I: Do you see how there are two columns?

S5: Yeah.

I: So one [column] was playing around with re-writing [33] as $11-1$, $12-1$ and then $10+2$ and you saw—you see that happens again $11-1$ and 12, and then $10+1$, and then somehow right, this person is just trying, just keeps trying it and then starts to see $5-1, 5$, and $5+1$.

S5: Mm hmm.

I: Right, do you see how they're just playing around? And then that was the key idea. That's the key idea that was synthesized from generating the examples and then you see the idea used in the proof. Ok? And you see the first idea? That this person used? It's crossed out. The $m-1, m-1$, and $m+2$? That 11? You see in the first example? $11-1, 12-1$ and $10+2$?

Transcript 2

I: What about Flexibility? What did you guys put for Flexibility?

S5: Advancing—er—developing.

Student 7 (S7): Developing?

Student 8 (S8): Advancing.

I: Why developing?

S5: 'Cause they tried two proofs.

S7: 'Cause she wrote proof by contrapositive, but just—

S5: Cancelled out.

S7: Yeah.

I: Ohh!! I didn't see that. Yeah, true.

Transcript 3

I: [To S5] What was it that you thought was interesting about this one?

S5: The fact that she changed it—well first that she made it—or he—I don't know who it is— $m+m+m$ and then made it $m-1$ and m and $m+1$.

I: Yeah, so did that, so did that, show up in your rubric somewhere?

Several students: Tools and Tricks.

I: Tools and tricks. Cool. Tools and Tricks and under [Between] Examples. Right, because that was generated, that key idea came from doing the examples.

Transcript 4

I: What did you guys put for Posing Questions?

Student 9 (S9): No questions.

S5: Yeah, I just put beginner.

Student 10 (S10): I don't think any of them had a question.

I: Yeah, so that's nothing. It shouldn't even—there's not a mark on it.

S5: Oh.

I: 'Cause there's no questions.

Transcript 5

I: So what did you guys have for Between Representations?

S10: I don't know what those—that—means? I don't know if anyone else feels that way.

I: Yeah, we can discuss it.

S5: Yeah, I confuse that with [Between] Examples sometimes. I noticed.

I: OK. Between Representations—yeah does anyone else have questions about it? Between Representations? Anyone have suggestions? Do you know what that means?

Student 11 (S11): I guess like in my head, representations are different ways to think about math. Examples are you're plugging it in. You're actually trying to find a—like an actual valid mathematical statement. So you kinda have more integers and stuff. And representations be can be like graphs, equations, they can be like models, they can be pictures however you want to look at it.

APPENDIX B: STUDENTS' WORK

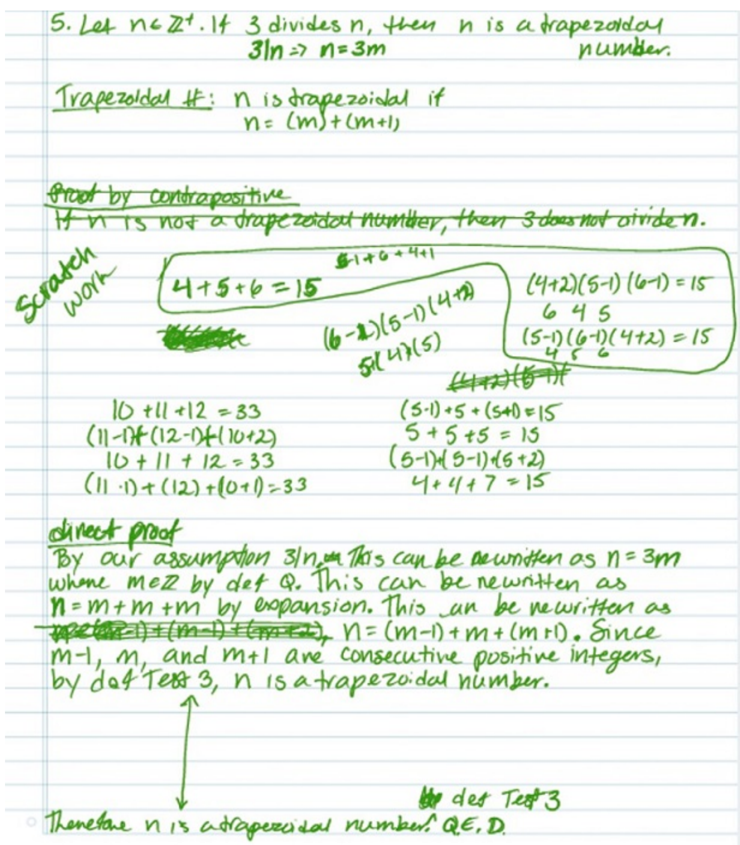
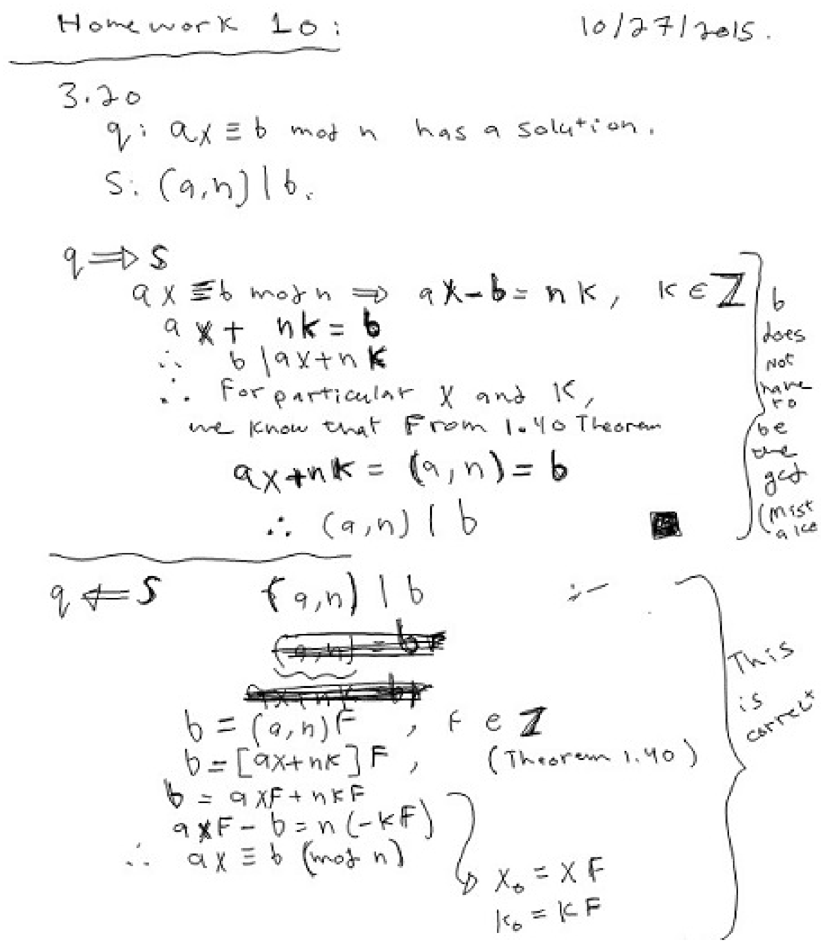


Figure 1. S4's work.



Homework 10: 10/27/2015.

3.20

q: $ax \equiv b \mod n$ has a solution.

S: $(a,n) \mid b$.

$q \Rightarrow S$

$ax \equiv b \mod n \Rightarrow ax - b = nk, \quad k \in \mathbb{Z}$

$ax + nk = b$

$\therefore \; b \mid ax + nk$

$\therefore$ For particular x and k,

we know that From 1.40 Theorem

$ax + nk = (a,n) = b$

$\therefore \; (a,n) \mid b$

b does not have to be the gcd (mistake)

$q \Leftarrow S \qquad (a,n) \mid b$

$b = (a,n)F, \quad F \in \mathbb{Z}$

$b = [ax + nk]F, \quad$ (Theorem 1.40)

$b = axF + nkF$

$axF - b = n(-kF)$

$\therefore \; ax \equiv b \pmod n$

$x_0 = xF$

$k_0 = kF$

This is correct

Figure 2. T5's work.

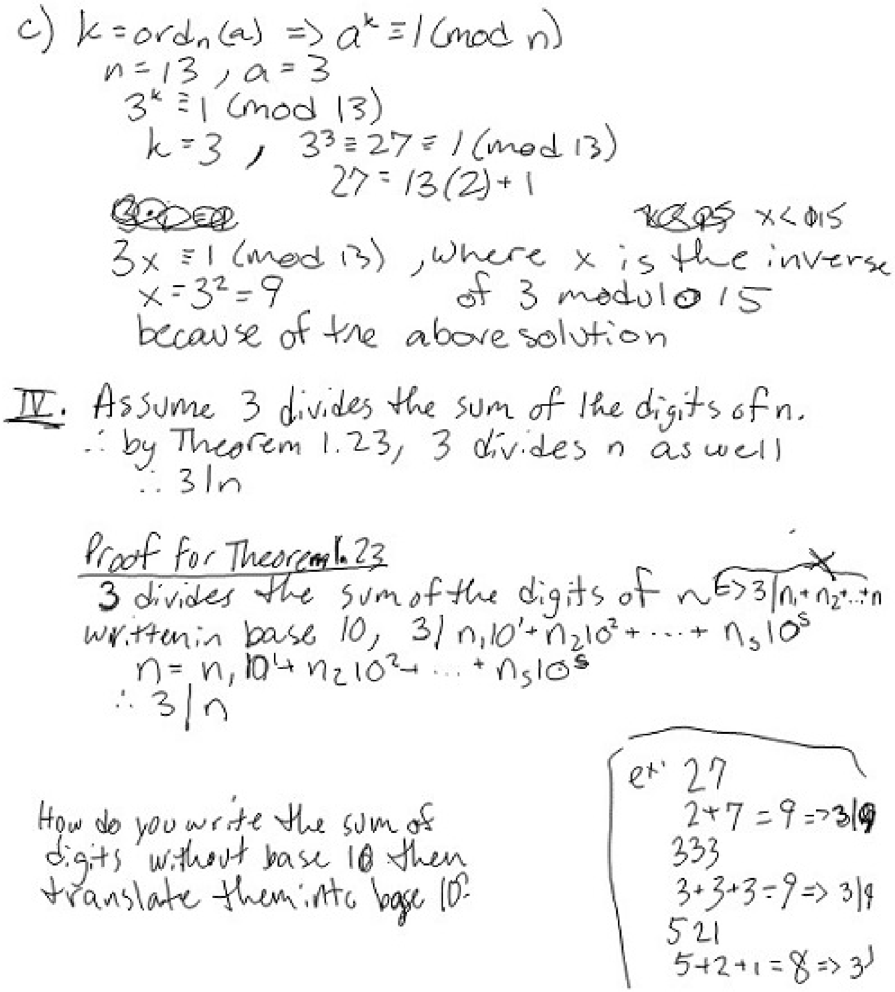
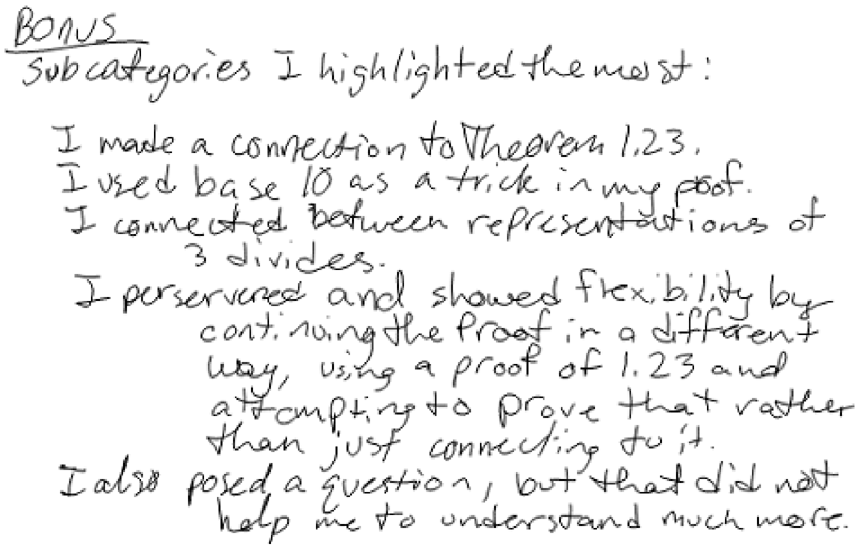


Figure 3. T4's work.



<u>BONUS</u>
Subcategories I highlighted the most:

I made a connection to Theorem 1.23.
I used base 10 as a trick in my proof.
I connected between representations of 3 divides.
I perservered and showed flexibility by continuing the proof in a different way, using a proof of 1.23 and attempting to prove that rather than just connecting to it.
I also posed a question, but that did not help me to understand much more.

Figure 4. T4's work.