CREATIVITY-IN-PROGRESS RUBRIC ON PROBLEM SOLVING AT THE POST-SECONDARY LEVEL

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Promoting students' mathematical creativity while problem solving is critical to prepare students for future learning and careers. In this paper, we introduce the Creativity-in-Progress Rubric (CPR) on Problem Solving as a tool to enhance mathematical creativity while cultivating problem-solving heuristics and fostering metacognition. With its two categories, Making Connections and Taking Risks, the CPR aims to develop mathematical discourse centered around aspects of creativity involving fluency, elaboration, flexibility, and originality.

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Mathematical creativity and problem solving are two interrelated research constructs in that "[t]rue problems need the extra-logical processes of creativity, insight, and illumination, in order to produce solutions" (Liljedahl, Santos-Trigo, Malaspina & Bruder, 2016, p.19). Numerous research studies and curriculum documents have emphasized the importance of mathematical creativity in mathematics and mathematics courses (e.g., Borwein, Liljedahl, & Zhai, 2014; CUPM, 2015; Leikin, 2009; Silver, 1997; Sriraman, 2009). Similarly, many research studies (e.g., Carlson & Bloom, 2005; Pólya, 1957; Schoenfeld, 2013) have emphasized the importance of problem-solving practices and identified a need to foster skills (e.g., metacognition, creativity) beyond accumulation of facts or procedural steps during problem solving. It seems that exploring mathematical creativity and problem solving together at the tertiary level in mathematics courses is rare (e.g., Zazkis & Holton, 2009). As a first step towards understanding ways to foster and enhance students' mathematical creativity at tertiary level, our research team designed a formative assessment tool, the Creativity-in-Progress Rubric (CPR) on Problem Solving that capitalizes on interactions between creativity and problem-solving constructs. In this paper, we introduce the CPR on Problem Solving and its development. We provide empirical examples from undergraduate Calculus 1 student interviews to illustrate potential benefits of using CPR.

Theoretical Background

In our work, we view mathematical creativity as a process of offering new solutions or insights that are unexpected for the student with respect to their mathematics background or the problems they have seen before (Liljedahl & Sriraman, 2006; Savić et al., 2017). In contrast to examining final products of those processes, this definition is process-oriented, providing a dynamic view of creativity rather than a static one. This definition also encompasses creativity relative to the student versus creativity relative to the field of mathematics (Leikin, 2009).

Our conception and development of the Creativity-in-Progress Rubrics (CPR) was guided by this operational definition of mathematical creativity and situated within two theoretical perspectives: *Developmental*, and *Problem Solving and Expertise-Based* (Kozbelt, Beghetto, & Runco, 2010). The primary assertion of the *Developmental* theory is that creativity develops over time, and the main

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focus of investigation is a person's process of creativity. This perspective also emphasizes the role of environment, in which interaction takes place, to enhance the creativity. The *Problem Solving and Expertise-Based* theory with the emphasis on the role of an individual's problem-solving process brings forth key concepts such as problems and heuristics.

In our work, we adopted Schoenfeld's (1983) formulation of a problem as a task that the problem solvers "don't know how to go about solving it" (p. 41). Thus, problem solving becomes a process in which the problem solver tries to attain some outcomes without having an immediate access to known methods (to that particular individual) (Schoenfeld, 2013). This description of problem solving aligns with our mathematical creativity definition as both of them focus on a process relative to the individual.

Creativity-in-Progress Rubric

In our previous research studies (see Creativity Research Group, n.d.), we explored the ways in which mathematical creativity could be explicitly valued and fostered in tertiary level proof-based mathematics courses. The CPR on Proving was rigorously constructed through triangulating research-based rubrics, mathematicians' and students' views on mathematical creativity, and students' proving attempts (Karakok et al., 2015; Savić et al., 2017; Tang et al., 2015). Following the development, the CPR on Proving was implemented as a formative assessment tool in several proof-based courses. Some instructors used it to facilitate in-class discussions on proof construction and evaluation of this process (El Turkey et al., 2018) whilst others gave it to students to be used on homework problems and write-ups of solutions (Omar et al., 2019). For example, one instructor, in an elective proof-based combinatorics course asked students to reflect on their proving process of assigned problems using the CPR. One of the students of this course, when asked to discuss the use of the CPR, stated "The reflection process – the rubric itself helped kind of outline where you should go if you were lost, in a very general sense." Another student said, "I think it's helped me …reflect on the sort of creative process that I have and it's kind of helped me understand the ways that I can be mathematically creative."

We have expanded our research program by modifying the CPR on Proving to problem solving by utilizing existing studies in problem solving. This effort allowed us to include more tertiary mathematics courses and student populations in our exploration of creativity. The CPR on Problem Solving has two categories: *Making Connections* (Figure 1) and *Taking Risks* (Figure 2). These categories are divided into subcategories that are reflective of the different aspects of creativity found in prior research. The rubric provides three general levels: Beginning, Developing, and Advancing, each of which serves as a marker along the continuum of a student's progress in that subcategory. This continuum among levels of the rubric communicates the possible states of growth, aligning with the theoretical constructs of the Developmental perspective.

Making Connections Category

The category of *Making Connections* is defined as a process of connecting the problem with definitions, formulas, theorems, representations, and examples from the current or prior courses and connecting the attempted problem solutions to each other. Various researchers (e.g., Schoenfeld, 2013; Silver, 1982) have highlighted the importance of prior knowledge in problem-solving processes acknowledging that such knowledge helps the problem solver to understand the problem and influences the choices of approaches and tools to be used (e.g., examples, representations). The subcategories in Making Connections communicate these ideas to the problem solver and encourage them to push their processes in these areas forward along the continuum. Furthermore, the Between Solutions subcategory encourages the solver to examine their different solution attempts, connect them, and generalize them for thorough understanding.

MAKING CONNECTIONS

	Beginning	Developing	Advancing
Between Definitions/Formulas/ Theorems NA ☐	Recognizes some relevant definitions/formulas/theorems from the course with no attempts to connect them in a solution	Recognizes some relevant definitions/formulas/theorems from the course and attempts to connect them in a solution	Uses relevant definitions/formulas/theorems from the course or other resources outside the course in a solution
Between Representations[1] NA ☐	Provides a representation with no attempts to connect it to another representation	Provides multiple representations and recognizes connections between representations	Provides multiple representations and uses connections between different representations
Between Examples NA ☐	Generates one or two specific examples with no attempt to connect them	Generates one or two specific examples and recognizes a connection between them	Generates several specific examples and uses the key idea synthesized from those examples
Between Solutions NA ☐	Attempts to connect multiple solutions to each other	Connects multiple solutions to each other	Connects multiple solutions to each other and generalizes common properties

Figure 1: Making Connections Category of the CPR on Problem Solving

This category encompasses the *fluency* and *elaboration* components of Torrance's definition of creativity (Leikin, 2009). As *fluency* describes flow of associations and use of basic knowledge, with its subcategories of between definitions, formulas, theorems, between representations, and between examples and continuum levels, Making Connections provides opportunities to enhance fluency. As *elaboration* relates to generalization of ideas, moving in rubric's the continuum toward advancing levels of each subcategory provides opportunities for generalization.

Taking Risks Category

The category of Taking Risks in our rubric is defined as a process of actively attempting a solution, demonstrating flexibility in using multiple solution paths, posing questions about reasoning within solutions, and evaluating solution attempts or solutions. The subcategories of Flexibility, Posing Questions, and Evaluation of Solution Attempt align with Pólya's (1957) problem-solving heuristic. In the third step of this heuristic, Pólya discusses the process of carrying out a plan and in the fourth step, the solver examines the reasoning and results of their solution attempt and tries to solve the problem in different ways. In addition, the continuum levels of the Posing Questions subcategory provide ways for the solver to move from the state of being stuck to less stuck by explicitly asking various types of questions.

TAKING RISKS

	Beginning	Developing	Advancing
Tools and Tricks[2] NA ☐	Uses a tool or trick that is usual for the course or the student	Uses a tool or trick that is partly unusual[3] for the course or the student	Creates a tool or trick that is unusual for the course or the student
Flexibility[4] NA ☐	Introduces one solution path	Introduces more than one solution path	Uses more than one solution path
Posing Questions NA ☐	Recognizes there should be a question asked, but does not pose a question[5]	Poses questions clarifying a step within a solution	Poses questions about reasoning within a solution
Evaluation of Solution Attempt NA ☐	Checks surface-level[6] features of a solution attempt	Checks an entire solution attempt for reasoning	Revises or validates an entire solution attempt for reasoning

Figure 2: Taking Risks Category of the CPR on Problem Solving

We note that the Tools and Tricks and Flexibility subcategories directly relate to the *originality* and *flexibility* components of Torrance's definition of creativity (Leikin, 2009), respectively. Torrance describes *originality* as a unique way of thinking, which could be evident in the process of using a trick (e.g., adding one and subtracting one) or introducing a mathematical object (e.g., defining a new function) that is unconventional for a student or a course that the student is in. Torrance defined *flexibility* as approaching a problem in multiple ways and producing multiple solutions, which is captured in our Flexibility subcategory. Within the Taking Risks category, we claim that the process of moving forward in the continuum of levels towards the advancing level requires a problem solver to take an intellectual risk in their problem-solving process.

Discussion

In our research project, instructors of Calculus 1 at several different institutions were asked to use the CPR on Problem Solving with tasks that we designed (El Turkey et al., in press). Each instructor decided how to implement these tasks and the CPR, where some used them as part of assignments and others had in-class sessions. We conducted interviews with students from these courses. In our preliminary analysis, we noted that students' experience and the usage of the CPR align with four themes of a problem-solving activity that Schoenfeld (2013) claimed to be necessary and sufficient for the analysis of the success of a problem solver's problem-solving attempt: a) The individual's knowledge; b) The individual's use of problem solving strategies, known as heuristic strategies; c) The individual's monitoring and self-regulation (an aspect of metacognition); and d) The individual's belief systems (about him- or herself, about mathematics, about problem solving) and their origins in the students' mathematical experiences.

We claim that the first two themes (a & b) directly relate to the CPR. When students utilize the CPR during their problem-solving attempt, they demonstrate their knowledge and use of problem-solving strategies. For example, one Calculus 1 student stated that the rubric prompted her to think about class work during problem solving. Discussing her required use of the CPR on an assignment during an interview, she said, "I was trying to think about the definitions we used in class and like drawing pictures with that" and continued by discussing that the flexibility and evaluation subcategories guided her problem-solving approach.

We believe the third theme (c) was encompassed by the usage of the rubric as a reflection tool as the problem solver tried to move forward on the continuum. The CPR connects to the fourth theme (d) as it may increase students' awareness and shift in their perception about their own creative processes (Cilli-Turner et al., 2019). For example, a student from another Calculus 1 course at a different institution stated that, "So, I feel like [the rubric has] definitely improved my creativity the way that …made me think a little bit more about what I'm actually writing down instead of just doing the problem." Our preliminary analysis seem to indicate that as a reflective tool, the CPR can help facilitate discussions on students' attempts and provide guidance on how to enhance students' mathematical reasoning and creative potentials. Ultimately, it may serve to make the link between problem solving and mathematical creativity more salient and accessible in any classroom context.

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