Chapter 13
"Creativity is Contagious" and "Collective": Progressions of Undergraduate Students' Perspectives on Mathematical Creativity

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Abstract: In this chapter, the Creativity Research Group shares results from a study in which progressions of undergraduate students' perspectives of mathematical creativity in an introduction-to-proofs course were explored. The course was intentionally designed to value students' creativity throughout in-class activities and out-of-class assignments. The Creativity-in-Progress Reflection (CPR) on Proving, a formative assessment tool, was introduced to students and used in several class sessions and assignments. In this phenomenological qualitative study, four participants were purposefully selected and their lived experiences with the phenomenon of mathematical creativity are presented to describe their progressions of their perspectives. Several data sources such as pre- and post-surveys, reflection assignments, transcripts of class sessions and interviews were utilized in the analysis. Results indicate that even though all four students had initially similar perspectives of mathematical creativity including unique, innovative, original approaches or different ways of solving a problem, their perspectives evolved to incorporate different mathematical actions that they utilized to be more mathematically creative.

Keywords (7-8): creativity-in-progress reflection, inquiry-based learning, proofs, introductory proof course, mathematical creativity, undergraduate mathematics, collective creativity.

Introduction

> It is shocking to me that the creativity involved in math is often overlooked...A professor at my school is dedicated to integrating creativity in her coursework to ensure students are engaged and excited throughout the entire course. It was in her course that my interest into the topic of number theory grew. (Peyton, a first generation, female student)

Peyton was one of the participants of the presented study and this short anecdotal quote was from personal communication a year after the study. As Peyton points out, mathematical creativity is often underemphasized in mathematics courses that tertiary level students take, even though there are numerous policy and curriculum-standard documents, both in the United States and internationally, emphasizing creativity as an important and needed skill when learning mathematics (e.g., Askew, 2013; Schumacher & Siegel, 2015). Peyton's quote also highlights the potential impacts of explicitly valuing and providing ways for students to enhance their mathematical creativity in mathematics courses. We, the Creativity Research Group, recognize the need to emphasize mathematical creativity and, in our research studies (e.g., Cilli-Turner et al., 2019 & 2020; El Turkey et al., 2018; Omar et al., 2019; Savić et al., 2017; Tang et al., 2015), we aim to explore the ways in which students' mathematical creativity can be explicitly valued and enhanced at the tertiary level mathematics courses. We agree with Nadjafikhah, Yaftian, and Bakhshalizadeh's (2012) claim that fostering mathematical creativity should be one of the goals of any education system.

The lack of a universally agreed-upon definition of mathematical creativity (Mann, 2006) should not prevent us from nurturing all our students' existing mathematical creativity in our courses. Since mathematics is so prevalent and acts as a gatekeeper in science, technology, engineering, and mathematics (STEM) fields, "[t]eaching engineers (and other STEM disciplines) to think creatively is absolutely essential to a society's ability to generate wealth, and as a result provide a stable, safe, healthy and productive environment for its citizens" (Cropley, 2015, p. 140). The number of studies examining students' mathematical creativity and the ways to enhance it at the tertiary level is slowly growing, however, compared to the number of studies at primary and secondary school mathematics level, it is still sparse. Furthermore, most of the studies focus on the quantitative outcomes of mathematical creativity; there is a need to understand the phenomenon of mathematical creativity through students' lived experiences in order to build classroom experiences to support it.

To complement the existing knowledge and expand our understanding of perspectives that students bring to mathematics, we share results from one of our studies conducted in an introduction-to-proofs course to explore the progression of undergraduate students' perspectives of mathematical creativity. This progression was examined through several data sources collected chronologically, including pre-course survey, students' reflection assignments, in class conversations, post-course survey, and one end-of-semester interview. Students' perceived development of their own mathematical creativity guided our understanding of these progressions. Using a phenomenological study design, we explore students' lived experiences of mathematical creativity in a proof course.

Background Literature

Guilford (1950), in his presidential address to the American Psychological Association, urged researchers and educators to find ways to enhance the creative promise of learners. However, from a research perspective, this has been a hard task as there are more than 100 definitions of mathematical creativity (Mann, 2006). In fact, Borwein et al. (2014) demonstrated that many mathematicians had different ideas about mathematical creativity. Some conceptualizations of creativity focus on emphasizing whether the end product is original and useful (Runco & Jaeger, 2012), while others describe mathematical creativity as a process that involves different modes of thinking, some of an unusual nature (Balka, 1974).

Most frequently, we observe researchers focusing on quantitative measures with an end-product orientation using the Torrance (1966) categories of *fluency, flexibility, originality,* and *elaboration* as a framework for data analysis. *Fluency* in general refers to the number of meaningful and relevant ideas as a response to a problem or a stimulus. *Flexibility* is defined as the number of groups or categories of responses, whereas *originality* (or novelty) is a unique production or an unusual thinking. *Elaboration* is defined as the ability to produce a detailed plan and relates to generalization of ideas.

Leikin (2013), for example, used a point system to evaluate three of these categories (fluency, flexibility, and originality) in students' work. While Leikin acknowledged that solutions must be "appropriate" – "The notion of appropriateness has replaced the notion of correctness" (p.391), an expert (e.g., an instructor or a researcher) was the one who judged what is or should be appropriate or original. Furthermore, even though quantitative approaches can measure how creative a product is from the perspective of an expert, they obscure completely the perspective of the student writing the solution and how others (e.g., students or instructors of the courses) may perceive it.

In one of our earlier studies, we explored university students' and mathematicians' perspectives of mathematical creativity using three process categories: taking risks, making connections, and creating ideas (Tang et al., 2015). We found that students rarely (9% of students' responses) associated making connections (e.g. synthesizing different mathematical content) with creativity when compared to mathematicians (38% of mathematicians' responses). This study provided motivation to think about explicitly valuing and discussing the processes that are deemed to be important in the existing literature to develop mathematical creativity (El Turkey et al., 2018). Furthermore, we recognize that most of the existing definitions of mathematical creativity were derived from experts' (e.g., research mathematicians, other experts in education and psychology fields) experiences or perspectives; however, it is (as) important to consider students' voices and experiences with mathematical creativity. Researchers have been calling for studies that bring students' voices forward (e.g., Roos, 2019 on the topic of inclusion), and it is time for us to hear undergraduate students' perspectives on mathematical creativity.

With this overarching goal to gain understanding of students' perspectives on mathematical creativity, we conducted the present study in an introduction-to-proofs course and examined the data collected at various stages of the course. Earlier results from this study used hypothesis coding (Saldaña, 2013) on students' end-of-semester interviews only and indicated that students' perspectives were related to Torrance's (1966) originality category with codes such as uniqueness, original, and innovative ways (Cilli-Turner et al., 2019). We also observed "being flexible" and "trying different ways" in students' perspectives, which related to Torrance's (1966) flexibility and fluency categories (Cilli-Turner et al., 2019). We further explored the sources of students' perspectives (Cilli-Turner et al., 2020) and noticed that students during these interviews mentioned their experience in the introduction-to-proofs course.

These results initiated our work here examining the progression of these students' perspectives of mathematical creativity from the start of the introduction-to-proofs course until its end. More precisely, we aim to address the research question: What are the progressions of students' perspectives on mathematical creativity through their experiences in a semester-long introduction-to-proofs course?

Theoretical Perspective and Methodology

In our mathematical creativity research projects (see http://www.creativityresearchgroup.com for complete list of references), we use a developmental perspective of creativity (Kozbelt et al., 2010) that contends that creativity develops over time and emphasizes the role of the environment in the development of creativity. Such an environment should provide students authentic mathematical tasks and opportunities to interact with others (Sriraman, 2005).

We operationalize mathematical creativity as "a process of offering new solutions or insights that are unexpected for the student, with respect to their mathematical background or the problems [they've] seen before" (Savić et al., 2017, p.1419). This definition focuses on the process (Pelczer & Rodriguez, 2011) of creation, rather than the product that is created at the end of a process (Runco & Jaeger, 2012). This orientation allows for a dynamic view rather than a static one to capture nuances in the individual's thinking and experiences. Furthermore, our definition takes a relativistic perspective—creativity relative to the student—in contrast to absolute creativity for the field of mathematics (Leikin, 2009). For example, Levenson (2013), using a similar viewpoint, focused on the discussion of ideas by individual students and how these ideas helped in developing a product of collective mathematical creativity in fifth- and sixth-grade mathematics classrooms. Levenson also emphasized the teachers' roles in facilitating these discussions. The developmental perspective of creativity interlaced with our operational definition of mathematical creativity is our theoretical perspective.

We utilized a phenomenological case study design (Patton, 2002) to explore the progression of students' perspectives on mathematical creativity through their experience in the introduction-to-proofs course. This methodology was suitable for our investigation as it is "particularly effective at bringing to the fore the experiences and perceptions of individuals from their own perspectives, and, therefore, at challenging structural or normative assumptions" (Lester, 1999, p.1). This allowed us to focus on students' perspectives of mathematical creativity rather than experts' (e.g., mathematicians' or researchers' view) views or normative assumptions.

Method

Setting

The research setting for this study was an introduction-to-proofs course at a small liberal arts college in southwestern United States. The course met twice a week for 15 weeks for approximately an hour and a half. The course topics typically included sets, logic, and various proof techniques (e.g., direct proof, induction, contradiction, contraposition). The instructor of the course implemented inquiry-based learning (IBL) pedagogy (see https://www.inquirybasedlearning.org/) and adapted Ernst's (2017) textbook. Students often worked in small groups and presented their proofs to the class, which was followed by class discussions. There were three exams during the semester and a final exam in week 16. Students had daily assignments with problem sets for which they had unlimited opportunities to re-work their proofs during the semester using feedback provided by the class community (peers and instructor).

Almost every week, students also submitted a reflection (short writing) assignment (RA). The content of these reflections varied from week to week. For example, for the first three assignments students were asked to read articles on topics such as importance of discussions in mathematics courses, importance of reflection, impact of IBL. Then, they wrote a minimum of a half-page reflection on what they learned and found meaningful. There was also one RA for each of three exams. For the RA in week 5 (RA#5), students were asked to reflect on their view of creativity and mathematical creativity and how these views were similar to and different from each other.

Students were introduced to the Creativity-in-Progress Reflection (CPR) on Proving (e.g., Karakok et al., 2016; Savić et al., 2017, named Creativity-in-Progress Rubric in our earlier publications) in class during week 7 (see Appendix 1 for the version used in this course). The CPR was developed as a formative assessment tool with two categories (making connections and taking risks) that incorporated aspects of fluency, flexibility, originality, and elaboration from existing research. The *Making Connections* category is defined as the process of connecting the proving task with definitions, theorems, multiple representations, and examples from both the current course, and possible experiences from previous courses. The *Taking Risks* category is defined as the process of actively attempting a proof to demonstrate flexibility in using multiple approaches or techniques, posing questions about reasoning within the attempts, and evaluating those attempts. The CPR also provides three general development levels: *Beginning*, *Developing*, and *Advancing*, each of which marks a student's progress on a given task along a continuum, as our way to communicate the possible states of growth.

The CPR was given to the students without the word "creativity" in the title to not steer students' attention to that word, but rather used the title "Progress Rubric on Proving" to focus on "progress" (see Appendix 1). Students were asked to reflect, using the CPR, on their proofs (including all scratch work) on various assignments and exams. In addition, the instructor asked students to use the CPR in class on their peer's presented work to discuss ways in which it demonstrated various aspects on the CPR as well as to engage students to think about how to move their own thinking forward to advancing levels on the rubric categories.

Participants

Fifteen students out of 17 enrolled students in the course agreed for their submitted course work and their utterances from class recordings to be used for the study. We conducted one, 60-90-minute audio-video recorded interview with seven participants who volunteered for these interviews. In this chapter, we present results from four participants to demonstrate the uniqueness of each participant's perspectives on mathematics and mathematical creativity while cross-case analysis resulted in common themes on what contributed to the development of their mathematical creativity, from their perspectives, in this course. All four students were female and first-generation students, and Table 1 summarizes the four participants' information.

Table 1

Four Participants Information

Participants (Pseudonyms)	Ethnicity	Major	Math Courses concurrently enrolled
Alice	Latine*	Mathematics	Number theory
Stephanie	White	Mathematics	Number theory; Calculus 3
Peyton	White	Economics (math minor)	None
Olivia	Latine*	Biology (math minor)	None

*See https://www.vox.com/the-highlight/2019/10/15/20914347/latin-latina-latino-latinx-

These four participants were selected among seven participants using both convenience and maximum variation sampling methods (Patton, 2002). With convenience sampling, we considered the most complete data set from participants to gain better insights into their lived experiences in the course. In addition, we examined all seven students' pre-and post-course survey entries, perspectives of mathematical creativity and progressions, students' background information (e.g., majors, courses taken), and self-identified mathematical ability for maximum variation. With maximum variation, we do not mean that the other three participants did not have any variation in their progression of perspectives; but rather with these selected four students, we were able demonstrate varieties in progressions for all seven students. In other words, a maximum variation (heterogeneity) sampling (Patton, 2002) was utilized to provide "high-quality, detailed descriptions of each case, which are useful for documenting uniqueness" and document "shared patterns that cut across cases" (Patton, 2002, p. 235).

Data Collection and Analysis

There were several data sources collected for the study: surveys, audio recording of class sessions, reflection assignments, and one 60-to-90-minute, audio-video recorded interview with seven students. Our first round of analysis of these data started by examining the interview transcripts from all seven students (Cilli-Turner et al., 2019 & 2020) which addressed the research question: What are tertiary students' perspectives of mathematical creativity?

We follow these results by using a case study with the selected students to further explore their experiences throughout the course. To address our research question for this study, the following data sources were organized chronologically: open questions on pre- and post-course surveys (collected during week 1 and week 15), reflection assignments: RA#5 on creativity and mathematical creativity; RA#8 and RA#12 on which students discussed their creative moments on exams 2 and 3, respectively, and any other available RAs, class discussions on the CPR (weeks 7 and 9), and the interview at the end of semester (conducted in weeks 15 and 16).

Students were given pre- and post-course surveys with the same questions. There were three open-ended questions where students typed their answers. For example, one of the questions was: To be good in math, you need to ….. because…. The other survey questions asked students to rate their perceived abilities in doing mathematics, attitudes about mathematics, and agreement with statements related to doing mathematics (e.g., Doing mathematics involves creativity; Taking risks is important in doing mathematics, etc.). Data from the surveys were included in the data analysis to gain insights to participants' views of mathematics in general.

Each student participated in one 60-to-90-minute audio-video recorded interview at the end of the semester. Interviews were transcribed in their entirety. At the beginning of the interview, all participants were asked what mathematical creativity meant to them with follow-up clarifying questions. We then asked students to expand on their RA#5, if they felt creative in the course, and to give a specific moment from the course as an example of their mathematical creativity. Similarly, they were asked if they thought other students were creative and asked to give examples. Additional questions were about what students thought contributed to their and other's creativity, and if and how they utilized the CPR in their work in this course.

Data analysis methods were chosen to fit the phenomenological study design: themes created to describe each student's experience throughout the course to understand the phenomenon of mathematical creativity and progression of their perspectives of it. Analyses sought "descriptions of what [students] experience and how it is that they experience what they experience" (Patton 2002, p. 107) related to mathematical creativity. This follows the theoretical perspective of the developmental creativity; each student's experiences in the environment (the course) will be different and progress based on what they perceive in the environment over time.

The chronologically organized data for each participant was read several times prior to start of the systematic process of coding. In these initial readings, the goals were to get a sense of what each participant uttered (as captured in transcripts) or wrote about their beliefs, skills, and perspectives on mathematics, creativity, and mathematical creativity by identifying relevant texts (Auerbach & Silver, 2003) which are texts (e.g., portion of a transcript or written work) that include words, sentences, or phrases related to research questions from all data sources. The first author turned these relevant texts into narratives using bracketing (Patton, 2002) to briefly describe the participant's experience with the phenomenon of mathematical creativity at that point in time. After narratives were created for each participant, the relevant texts were coded further into ideas that were repeated by the participants at least three or more times, and participants' own words or phrases were used to describe these codes (Auerbach & Silver, 2003).

For example, some repeated ideas were metaphors students used, "thinking outside the box," or words, "different ways," "making connection," and so forth. These repeated ideas were checked with the results of the earlier studies, Cilli-Turner et al. (2019 & 2020), for triangulation purposes (Patton, 2002). Then, we looked for occurrences of the repeated ideas for each participant to gain insight into the progression of their perspectives on mathematical creativity.

The first author continued with examining the narratives to add more nuances related to the participants' experiences with the repeating ideas. These revised narratives were re-examined and re-written (process described in van Manen, 1990) to capture overarching themes for each participant's experience. One important aspect of our data analysis method was to examine each participant's perceived development of their own mathematical creativity. The mathematical actions (e.g., taking risks, posing questions, etc.) that participants used when explaining their mathematical creativity (e.g., "I made connection to…") in RA#8, RA #12, and during the interview, provided us additional insights into participants' operationalization of, and thus, their conception of mathematical creativity. In other words, such self-reflections of ability brought "to the fore [the students'] experiences and perceptions" of mathematical creativity "from their own perspectives" (Lester, 1999, p.1).

Analysis of pre-and post-course survey responses provided corroborating evidence of progression of participants' perspectives. Even though open-ended survey questions were not asking about mathematical creativity, participants' responses captured their initial and end of the course views related to mathematics; because "it has long been accepted that we understand new phenomena [of mathematical creativity] in terms of the understanding we already possess" (Spangler & Williams, 2019, p.4).

Results

We first present each participant's perspectives of mathematical creativity and progression of them through our developed narratives. Then, results of cross-case analysis are presented to share uniqueness and similarities of perspectives and progressions of four participants' perspectives. We start with Alice's narrative and then present condensed versions (due to space limitations) of Stephanie, Olivia and Peyton's narratives. Alice's narrative was provided in a longer version to exemplify the chronological data analysis process only. In the condensed versions, we provide participants' initial perspectives captured in RA#5 and address our research question on the progression of perspectives, by making references to the repeated ideas.

Progression of Alice's Perspective

Alice initially described her view of being creative in mathematics by referring to "finding different and innovative ways to come to a solution or a number of solutions" (RA#5). At this point, she associated creativity with the idea of "memorization" in mathematics: "It may also help to use creativity to help remember theorem[s] or formulas such as creating a song to remember the quadratic formula." This memorization idea aligns with her rating of the statement "the best way to do well in math is to memorize all the formulas" on the pre-course survey. Alice was the only student (amongst the seven) who slightly agreed (rated 4 out of 6 on the agreement scale) with this statement, whereas other participants disagreed with this sentence in varying degrees (i.e., strongly to slightly disagree).

In this initial perspective of mathematical creativity, Alice also mentioned that being creative in mathematics helps to understand concepts "easier," which, for her, seemed to be about creating ways to memorize formulas or methods of solution. For example, in week 7 during a small group discussion in which students were asked to comment on other students' RA #5 entries (which were presented without any student name), Alice shared her view of being mathematically creative as "not necessarily like making it colorful or pretty, just being able to find a different way to approach it or maybe a shorter way or a simpler way" (week 7, classroom transcript). In this discussion, she said, "That's what tripped me up when I started to get to college cause it's like 'Oh yeah! Here's all the ways you can do it!' and I'm like, 'there's that many ways? Really!?'" Her remarks made her group members laugh and one of them said, "Let's just stick with one!" Alice continued, "One! I can memorize one, I don't know if I can memorize 5." It seems that the idea of having multiple ways to solve problems was a new and challenging experience for her in college mathematics courses, and she related this experience to mathematical creativity at this point in the semester.

After the CPR was introduced to students, Alice seemed to relate her own proof construction process to various subcategories of *making connections* category the CPR (repeated idea). On both exam 2 and exam 3 reflection assignments (RA#8 and RA#12, respectively), for example, she mentioned trying to connect a theorem to the proof statement. "I was being creative when I tried to connect a previous theorem on the test to help prove another theorem on the test" (RA#8). As the repeated idea of making connections were observed in data after the CPR was introduced, we claim that it became part of Alice's perspective of mathematical creativity. It also seems that Alice started to incorporate the CPR language to examine her own work and found this practice (making connections) useful for her to better understand the course.

The end of the semester interview provides additional evidence for progression of Alice's perspective from "different and innovative ways" to making connections. We observed other aspects of her perspective that were not repeated or uttered by Alice prior to this interview. Alice described creativity in mathematics as

> coming up with like new and different techniques to be able to solve um problems… to be able to prove theorems, specifically for [this course]. Um, it kind of means just, kind of using like a trick um something that's not really common, or maybe like a different representation to show the same thing that no one has really used. (Interview, week 16)

She, yet again repeated the finding different ways ("different techniques") idea, but included using tricks or different representations in her description. For Alice, "using a trick" mathematical action (process) was related to making connections between theorems and the proof statement, which helped her to understand a proof more easily.

> I guess finding kind of like a trick or …being able to find the connections between theorems or being able to use one theorem to solve another or using like a lemma to solve part of a theorem, just to make it a lot more…easier so the theorem's not…a page and a half long…[I]n any case, just having, being able to find… a technique that works that doesn't necessarily make everything longer. It kind of just makes it more…easier to understand too. (Interview, week 16)

As Alice's perspective progressed to incorporate making connections, her view of the function of different ways or approaches moved away from memorization of formulas for understanding. For instances, Alice strongly disagreed with the post-course survey statement, "the best way to do well in math is to memorize all the formulas" with which she was in slightly agreement at the beginning of the course.

In addition, Alice "tried to feel" creative and she did feel creative through her attempts to make connections, even though she identified herself "struggling a little bit trying to make connections between um theorems, um and being able to like create lemmas to be able to uh fit into proofs." However, when she indeed made connections, she perceived this part of her understanding, "when I make a connection I'm like 'Yes! Like I understand'. Like it makes me really happy when I'm able to make a connection" (Interview, week 16). It seems that making connections not only helped her feel creative but it impacted her emotion ("makes me really happy").

Alice's perspective of mathematical creativity is centered around the idea of finding different solutions. Throughout the course, Alice incorporated strategies (e.g., using different examples, representations, etc.) to find these different solutions. She believed that her mathematical creativity ability developed when utilizing these strategies. She not only mentioned *making connection* repeatedly but she internalized these mathematical actions for her own mathematical creativity. For this reason, we claim Alice's perspective of mathematical creativity progressed to include making connections. It seems that at the end of semester, Alice recognized the ways in which making connections provided her the tools to improve her own mathematical creativity as well as to understand the concepts "better" and "easier" that were not (just) memorization of formulas.

Progression of Stephanie's Perspective

Stephanie's perspective of mathematical creativity centers around the metaphor of "taking the road less travelled" (repeated idea) that makes the most sense to the individual. This perspective progressed throughout the course to recognize the interplay between individual's mathematical creativity and collective mathematical creativity. Stephanie initially explained her perspective of mathematical creativity in terms of using "imagination to innovate something original" (RA#5) in an arts setting. However, in this reflection, she acknowledged that creativity can be found in other contexts and real-life situations. She claimed that mathematics professors "tend to teach the road most travelled to get to the solution, but more times than not, there are other ways to get to the correct solution," (RA#5) and, for her, finding these other ways to get answers was being mathematically creative.

Stephanie referred to a person's creativity relating to their own process of finding different solutions (repeated idea) and taking paths that were less travelled (repeated idea) again after week 5. However, when these ideas were repeated, she included the purpose of taking such different paths as personal sense making and understanding of mathematics. Furthermore, for her, determining the less travelled path required her to see other's paths. For example, when students were asked to reflect on their creative moments on exam 3 (week 12), Stephanie wrote:

> It is hard to determine if there were any 'creative moments,' as for me, creativity is the path less travelled. I do not know how my classmates proved any of them and so I don't know if any of my proofs were creative.

During the interview, Stephanie first described what it means to be mathematically creative as "the same as being creative in anything else. It's taking the road less traveled [repeated idea]. It's not just doing what the herd is doing but finding your own way to get to where you need to be." She again emphasized that the importance of sense making and understanding during the process of finding solutions, "It's finding the solution but doing it in a way that makes the most sense to you."

As she identified her and classmates' creativity to be developed throughout the course, she mentioned that seeing other's work in class was an important aspect of the course that contributed to these developments. When she elaborated on this idea, she stated that for her "creativity is both individual and collective…[another student's] creative moment, I could then use to expand on and do something a little different with to have my own creative moment." She believed that all of these creative moments of students were "not the road most travelled" and she viewed these moments to be an integral part of the "road we are traveling together, and yet each time we're changing it to be what we need it to be, expanding on it and having our own creative moments, based on a creative moment somebody else had before us."

For her individual creativity, she focused on the subcategory of tricks and tools of the CPR "to create something new…to have that one thing that's like 'Wow, that's awesome!'" She also wanted to do this for the collective creativity of the class, she wanted to "bring forth a new tool that we could all use as a class." She reflected in the interview that she "started to look at creativity a little bit different through the course" as she noticed many different "paths" the other students were taking both in their choice of proof technique and using conceptually different ideas.

Even though Stephanie repeatedly referred to the metaphor of "taking the road less traveled" to describe her perspective of mathematical creativity, her desire to compare her "road" to other's was not only to assess her own creativity but also for her to develop her creativity through other's creative moments. She believed that finding tools and tricks and sharing these with others, persistence, and being flexible to try different things were important for all of them to develop their individual and collective creativity.

Progression of Peyton's Perspective

Peyton's perspective on mathematical creativity progressed throughout the semester from believing that "there's no need for creativity in mathematics" to recognizing that mathematical creativity is within the process of producing proofs. Peyton described creativity to "be able to come up with original and innovative ideas" (repeated idea) initially and identified herself not creative but wished to be (RA#5). She mentioned in RA#5 that prior to the introduction to this proofs course she was on the “spectrum that generally believes that there’s no need for creativity in mathematics.” She enjoyed mathematics initially because she could get to the correct answer if she understood the material. This course had proven her initial ideas to be untrue and “every time I see someone else’s answer to a proof I am amazed at how he or she came to that answer…Overall I realized math has a lot of room for creativity” (RA#5).

Peyton’s perspectives on creativity started to evolve at the very beginning of the course (as evidence in RA#5) and she described her view of creativity in the interview (week 16) through a process that involves reflection in thinking, “[b]ecause every step requires more thinking, and every step requires you to figure out your next step, and so that’s where the creativity comes in.” We notice additional progression of Peyton’s perspective on mathematical creativity prior to the end of semester interview. For example, when she reflected on a quote given for the RA#8, she shared her noticing

> that creativity does not necessarily need to be a “spontaneous” and brand new discovery, because creativity can be presented in many ways. This is especially significant for me because I generally assume that creativity does in fact require spontaneous and new discoveries…But reading [the] quote emphasizes the fault in that logic. True creativity lies in person’s ability to use resources in ways to improve on his or her own thought process.

Even though Peyton did not perceive herself as mathematically creative throughout the semester, she was able to recognize other students’ mathematical creativity and hence thought she probably had mathematically creative moments as well. She believed that her mathematical creativity developed in this course, in particular “it helped me to realize or recognize that, that there does not always have to be one set process in math" (Interview, week 16). Overall, Peyton's perspective of mathematical creativity progressed from the ability "to come up with original and innovative ideas" (RA#5) spontaneously to recognizing that it is a process where "true creativity lies in person's ability to use resources in ways to improve on his or her own thought process" (Interview, week 16).

Progression of Olivia's Perspective

Olivia's perspective of mathematical creativity centers around the metaphor of "thinking outside the box" which she initially associated with "trying something new that is often different from others" (repeated idea of uniqueness) (RA#5). Throughout the semester, her perspective evolved to include other aspects of mathematical creativity related to the mathematical actions described in the categories of the CPR. She believed that the creativity was contagious and, through course discussions of their (students') proofs, thinking processes behind such proofs, and the CPR contributed to her perception of creativity and the development of her own mathematical creativity. At the end of the semester, she described mathematical creativity as "really thinking outside of the box [repeated idea] and being able to be comfortable or at least willing to take risks. And, not just follow a standard format… but being willing to be flexible and try different approaches."

She initially thought of creativity in art-related contexts and believed she did not have any of those qualities and hence never thought of herself as creative. However, in this course, she noticed that "there are mathematical ways of being creative, so I was able to get a better understanding as the semester went on of what that [creativity] meant in a different context" such as mathematics and proving. She noticed that students in the course developed their mathematical creativity, including her. She mentioned that early in the semester, they (students) were “kind of not really feeling confident in our abilities to be creative” however, later in the semester “it was really interesting to see students that were quiet, reserved early on, like show their work later in the semester and they had done something like totally cool and amazing.”

Olivia mentioned many aspects of the course contributing to her and her classmates’ development of mathematical creativity. For example, the IBL structure of the course was an important aspect because “you to really try to make connections and it forces you to get creative because you have, um, very little like understanding of the right way to do it, so it kind of throws that out of a student’s mind.” She viewed not having pre-exposure to the “right way” allowed them to be “free” and working with other students helped to develop different methods to approach problems. She said these course activities aligned with her idea that “creativity can be contagious.” Active class engagement in which students share their work and discuss their mathematical thinking was a way that Olivia thought the creativity spread out among students and developed their mathematical creativity.

Olivia’s beliefs about mathematics in general also showed some changes from pre- to post-course survey. On the pre-course survey, she stated that to be good in mathematics, one needs to “think outside the box [repeated idea] and be comfortable with abstract thinking because math is not one dimensional.” To her, thinking mathematically meant that “to think critically and use what I know and apply it to any given situation.” On the post-course survey, Olivia explained being good in mathematics through flexibility, persistence, creativity and the process of evaluation. These shifts from pre- to post-course survey served as corroborating evidence for progression of Olivia’s perspective of mathematical creativity.

Overall, Olivia internalized several mathematical actions from the CPR to help her "to think outside the box" and be mathematically creative: willing to take risks, being flexible, making connections and posing questions to further her thinking.

Uniqueness and Similarities in Progressions across Participants

All four participants' progressions of their perspectives of mathematical creativity had some unique aspects from their experiences in the course. Alice's perspective of mathematical creativity was centered around the idea of finding new, different techniques and solutions. Throughout the course, Alice incorporated strategies (e.g., using different examples, representations, etc.) to find these different solutions. She believed that her mathematical creativity developed utilizing these strategies, which was unique in her experience. For Alice, creativity is "being able to find the connections" to make mathematics easier to understand.

Stephanie's perspective of mathematical creativity was centered around the metaphor of "taking the road less travelled" and progressed to include the importance of others' contributions to have a collective creativity. Stephanie identified the subcategory of tricks and tools of the CPR as an important strategy to develop individual and collective creativity, which was unique to her experience. For Stephanie, creativity is "taking the road less traveled" individually and contributes to the "the road we are traveling together" to create collective creativity.

Peyton's perspective of mathematical creativity progressed from believing that "there is no need for creativity in mathematics" to view it as the ability "to come up with original and innovative ideas" spontaneously to recognition of it as a process where "true creativity lies in person's ability to use resources in ways to improve on his or her own thought process." Peyton mentioned many aspects of the course contributing to her recognition of mathematical creativity and development of mathematical creativity of others. Peyton's experience was unique in the way that she started to recognize many aspects of the course and continuously reflect on these experiences to integrate them in her perspective. For Peyton, creativity is finding your own ways and about the process of "getting from the beginning to the end" within the idea production.

Olivia's perspective of mathematical creativity centered around the metaphor of "thinking outside the box" and identified many strategies such as taking risks, being flexible and posing questions to develop mathematical creativity. As she believed that the creativity was contagious, posing questions to understand her and other's thinking was important to her. She wrote her own questions on her scratch paper, which was a new practice for her. In addition, she repeatedly talked about flexibility in terms of working on and finding many different solutions, which was another unique aspect. For Olivia, creativity is "contagious" and requires one to be "comfortable or at least willing to take risks."

There were similar repeated ideas in all four students' experiences with the phenomenon of mathematical creativity. Some of these repeated ideas were about aspects that contributed to the development of their mathematical creativity: course structure (e.g., IBL format) and use of the CPR, and the instructor's actions.

The IBL format of the course was new to all four participants and in this course, students actively engaged in constructing their own proofs and sharing their mathematical thinking, processes, and proofs with each other. All students mentioned in this course structure, they were not pre-exposed to the "right way" or "one way" of proving and they had to do what they thought "made sense to them" or "was going to work." They had to "swim or sink" and they were all in this together, so "you start to work together, and you start to build relationships with [classmates] and you work off of each other's creativity." Thus, this format helped students develop not only their proving skills but it also developed their mathematical creativity.

Examining each other's proofs was helpful to all four participants for different reasons. Alice valued this practice because it helped her to reflect on her own process to make sure she understands the mathematical concepts. She appreciated the use of the CPR in this process as it helped to notice the use of strategies mentioned on the CPR. For Stephanie, examining other's proofs was important to see if she "took the road less traveled" and to learn what other tools and tricks that she could use in her future proof work. However, she felt uncomfortable to use the CPR as she felt like she was being "judgy" (Week 9, classroom transcript). Olivia, and other students (including Alice), pointed out that the purpose of using the CPR was to understand each other's thinking processes. Examining proofs "opened" Peyton's "eyes to realize that there really are so many different ways to go about doing things, especially in math," and for Olivia, this was the essence of mathematical creativity and helped her to develop her own.

All four students noticed certain instructor actions that provided them opportunity to develop their mathematical creativity. Peyton considered the instructor's guidance and determination not to interfere with their learning process to be important. Olivia noticed that the instructor created a "safe", "free-spirited" environment that allowed them to take risks. She noticed even small actions by the instructor such as sitting down to students' table to join the conversation helped create this environment and facilitated the message "you know you guys kind of run the show type of deal" (Interview week 16). Stephanie thought the instructor's persistence of not giving out answers or confirming correctness was an important action for them to develop their mathematical creativity collectively. She acknowledged the instructor's intentions for a "swim or sink" approach as pushing them to focus on processes by guiding them through questions.

Conclusion

The purpose of this chapter was to present students' perspectives of mathematical creativity and how such perspectives develop in a course environment. The developmental orientation of creativity in this phenomenological study provided us the opportunity to hear student voices and notice the ways in which they experienced the phenomenon of mathematical creativity in the classroom environment that was carefully designed by the instructor who made explicit choices. The IBL implementation coupled with the use of CPR provided students opportunities to experience multiple ways of thinking, examining proof processes, and developing individual strategies. They internalized these experiences as part of mathematical creativity and utilized them to enhance their own mathematical creativity.

The results of this study contribute to our existing knowledge of creativity in several ways. The instructional designs (e.g., the course structure, teaching actions) can nurture students' perceived self-abilities of mathematical creativity and shape (progress) their perspectives of mathematical creativity. The CPR provides strategies for students to develop their own mathematical creativity in unique ways and provides additional tools to understand mathematics. The instructors' actions not only motivate students to form a classroom learning community but develop collective creativity. All these carefully engineered instructional efforts made creativity to be contagious.

The research design allowed us to break free from normative assumptions of mathematical creativity. We (the researcher and the instructor) purposefully did not evaluate students' mathematical creativity ability, rather attended to participants' voices to understand the formation of their perspectives. As we believe both the ability and perspectives of mathematical creativity are dynamic and shape continuously, our results only present the progressions of participants' perspectives at the time of the study. In the opening anecdotal quote, we have a glimpse of Peyton's perspective a year after the study. In this research study design, we also incorporated students' views of mathematics (through examining their pre- and post-course survey entries) as these perspectives intertwine with views of mathematical creativity.

In our current studies, we are exploring some of these experiences further. For example, we have been expanding on explicit instructors' actions to enhance mathematical creativity in Calculus courses (Tang et al., 2020). As discussed in this chapter, we noticed that students' utilized the CPR not only to develop their mathematical creativity but also to better understand the mathematics. We are exploring this connection with Calculus students' lived experiences (Cilli-Turner et al., forthcoming).

We also observed many participants of this study connected their experiences with emotions that they felt in the course with the phenomenon of mathematical creativity (e.g., Alice: "makes me really happy", Stephanie's excitement "Wow!"). This line of observations led us to focus on affective domains in mathematical creativity in our current work as well (Cilli-Turner et al., forthcoming).

In closing, we invite both researchers and instructors to design environments for students to notice their mathematical creativity in their own way and spread it to others. The developmental perspective of creativity provides us a roadmap for the "road we are traveling together" to understand the research construct of mathematical creativity "collectively" (Stephanie, week 16).

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Appendix 1

Progress Rubric on Proving

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MAKING CONNECTIONS:	**Beginning**	**Developing**	**Advancing**
Between Definitions/Theorems	Recognizes some relevant definitions/theorems from the course or textbook with no attempts to connect them in their proving	Recognizes some relevant definitions/theorems from the course and attempts to connect them in their proving	Implements relevant definitions/theorems from the course and/or other resources outside the course in their proving
Between Representations[1]	Provides a representation with no attempts to connect it to another representation	Provides multiple representations and recognizes connections between representations	Provides multiple representations and uses connections between different representations
Between Examples	Generates one or two specific examples with no attempt to connect them	Generates one or two specific examples and recognizes a connection between them	Generates several specific examples and uses the key idea synthesized from their generation

[1] We define a *mathematical representation* similar to NCTM's (2000) definition. It includes written work in the form of diagrams, graphical displays, and symbolic expressions. We also include linguistic expressions as a form of lexical or oral representation. For example, a student can use the lexical or oral representation, "the intersection of sets A and B"; a Venn Diagram to depict his/her mathematical thinking; a symbolic representation $A \cap B$; or set notation $\{x | x \in A \text{ and } x \in B\}$ (which is also a symbolic representation). Note the last two representations are in the same category, e.g. symbolic, but they are still considered two different representations.

Progress Rubric on Proving

TAKING RISKS:	**Beginning**	**Developing**	**Advancing**
Tools and Tricks[2]	Uses a tool or trick that is algorithmic or conventional for the course or the student	Uses a tool or trick that is model-based or partly unconventional[3] for the course or the student	Creates a tool or trick that is unconventional for the course or the student
Flexibility[4]	Begins a proof attempt (or more than one proof attempt), but uses only one approach	Acknowledges and/or uses more than one proving approach, but only draws on one proof technique	Uses more than one proof technique
Posing Questions	Recognizes there should be a question asked, but does not pose a question[5]	Poses questions clarifying a statement of a definition or theorem	Poses questions about reasoning within a proof
Evaluation of Proof Attempt	Examines surface-level[6] features of a proof attempt	Examines an entire proof attempt for logical or structural flow	Examines and *revises* an entire proof attempt for logical or structural flow

[2] Based on the Originality category from Leikin (2009).
[3] Learned in a different context.
[4] A proof attempt is a continuous, sustained line of reasoning focused on a single theorem or conjecture. A proof approach is a proof attempt in which a new or different (to the prover) idea is introduced. Finally, a proof technique is a proof approach that addresses the overall logical structure of the proof. Common proof techniques include induction, proof by cases, direct proof, contradiction, and contrapositive.
[5] For example, a student writes a "?" next to something.
[6] Surface-level features include technical, computational, and line-to-line logical details.