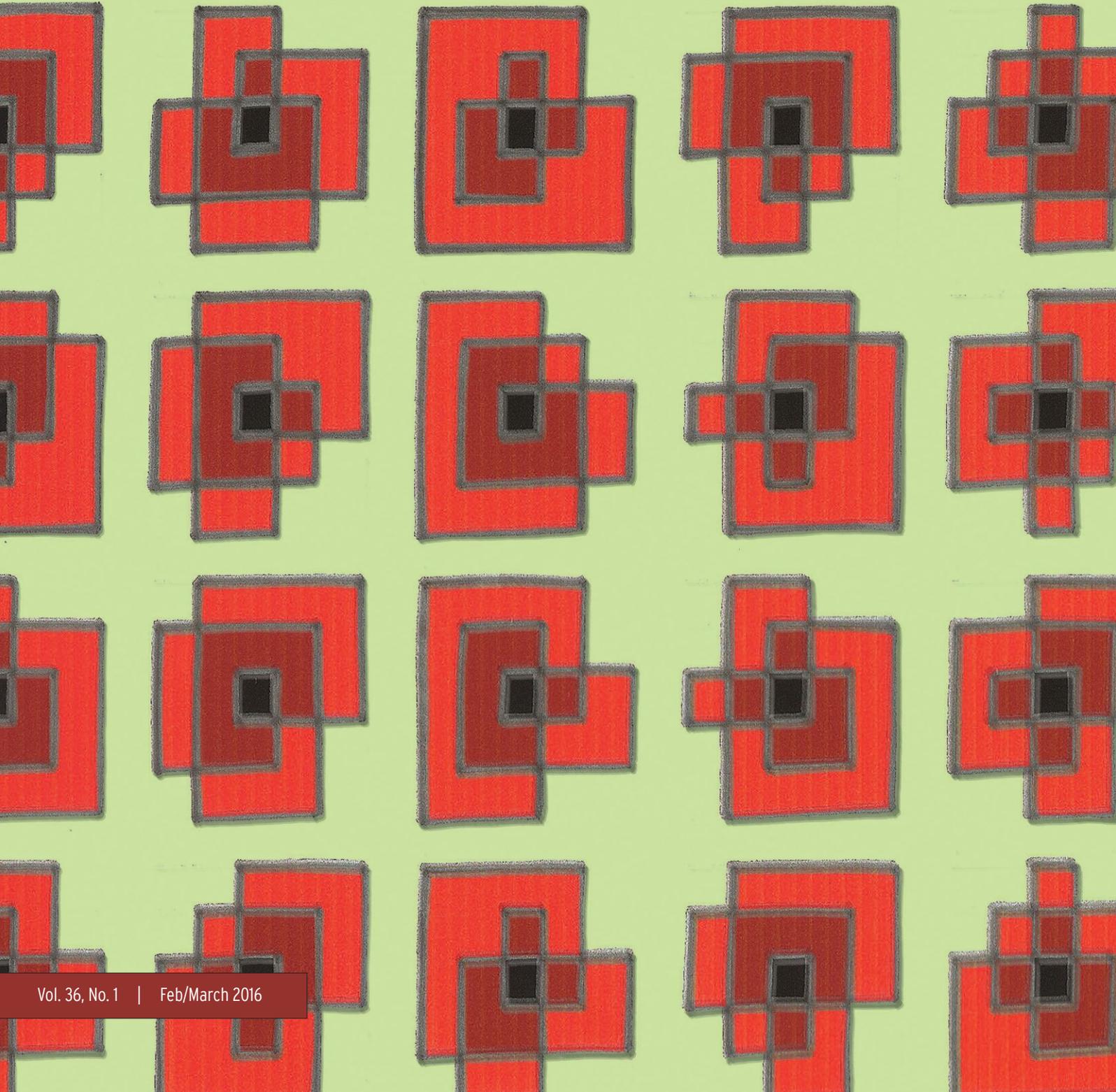


# MAA FOCUS

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## From the Editor



**If you are like me**, you have recently gotten back from the Joint Mathematics Meetings (and almost 6,300 people are in that category! What a great meeting!) and are getting settled into a new semester. This issue lets us reflect on JMM, with coverage of the sessions and events in Seattle.

It was a full meeting that gave me lots to think about. In addition to attending sessions focused on teaching

and my research area, I was inspired by Katherine Crowley (p. 9), who invites us to consider how we can implement change by becoming policy-makers. Perhaps, like me, you are concerned about funding for education and the future of STEM education, but you haven't thought about how to be involved.

On the lighter side, I continue to be charmed by Pig and Penguin (on my shoulder in the picture), who travel with James Tanton. See more photos of these two and other scenes from the Seattle meetings online at [maa.org/maa-focus-supplements](http://maa.org/maa-focus-supplements) and [flickr.com/photos/maaorg/](http://flickr.com/photos/maaorg/).

This issue also allows us to get back to our normal routine and remember how MAA and *MAA FOCUS* help us in our day-to-day mathematics careers. In particular, I'm thrilled to be continuing our new Toolkit column. We've received many contributions to this column, and I'm excited to share the advice and suggestions of other MAA members about teaching, research, service, and academic life.

We are also launching two columns in which we feature activities by sections (the North Central section in this issue) and SIGMAAs (this time BIG SIGMAA, and Stat-Ed). If your section or SIGMAA would like to be featured, please contact me at [maafocus@maa.org](mailto:maafocus@maa.org).

Moreover, we have some great reminders of how much fun it can be to be a professional mathematician, statistician, or mathematics educator—our columns are full of contributions from people who obviously enjoy their work and interactions with students.

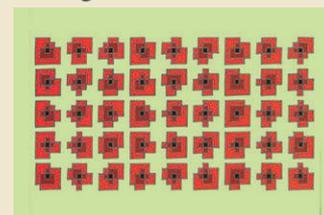
Share your joy and love of mathematics by submitting an article, column, or question for Dear MAA. Or snap a photo of math in your everyday life and send it to "Found Math." Our strength is in our membership, and we hope to hear from you soon at [maafocus@maa.org](mailto:maafocus@maa.org).

*Jacqueline Jensen-Vallin*

### ON THE COVER

One of the two winners at the art exhibition held during the 2016 Joint Mathematics Meetings:  
 45 Poppies by Karl Kattchee.

Table of Contents photo credits:  
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## TOOLKIT

### A Rubric for Creativity in Writing Proofs

—GULDEN KARAKOK, MILOS SAVIC, GAIL TANG, HOUSSEIN EL TURKEY, DAVID PLAXCO, EMILIE NACCARATO

**Given that creativity** is an important aspect of professional mathematicians' work, we want to make it explicit in our courses and, more importantly, value students' creative thinking. In proof-based courses, we commonly evaluate students' valid lines of reasoning and final proofs. However, we believe it is equally important that students understand the value of "play time" during proof construction—making connections, taking risks, and engaging in self-reflection during or after the proving process. Our formative assessment tool, the Creativity-in-Progress Rubric (CPR) on Proving, features these areas that help develop students' mathematical creativity.

#### Rubric Information

We developed the CPR on Proving after conducting interviews with research mathematicians who teach tertiary mathematics courses, and we incorporated results from existing research studies in mathematical creativity. The rubric has two categories: Making Connections and Taking Risks.

These two categories are divided into subcategories reflecting the different aspects that enhance the development of creativity found in prior research. We view Making Connections as the ability to connect the proving task with definitions, theorems, multiple representations, examples from courses that a student is in, and (possibly) experiences from

previous courses. The three subcategories (see figure 1) are designed to have students explicitly think about making connections during the proving process. In other words, this category aims at developing students' *fluency* and enhances transfer of learning.

#### The emphasis is on the proving process and the development of practices that mathematicians report can lead to creative thinking.

We define the second category, Taking Risks, as the ability to actively attempt a proof, perhaps using multiple proof approaches and/or techniques, posing questions about reasoning within attempts, and evaluating those attempts. Four subcategories (Tools and Tricks, Flexibility, Posing Questions, and Evaluation of a Proof Attempt as in figure 1) aim to push students' thinking further and have them persevere during proof construction. In other words, the intention of this category is to develop originality, flexibility, and elaboration.

For every subcategory, the rubric provides three levels: Beginning, Developing, and Advancing, each serving as a marker along the continuum of a student's progress in that subcategory during proof production.

#### Use in Proof-Based Courses

Both instructors and students can use the CPR on Proving. For instructors, it helps to explicitly value creativity

during the proving process by providing feedback to students in their proof construction. For students, the CPR on Proving helps guide their overall process of proof construction, including all of their scratch work (i.e., play time).

It is important that instructors communicate the idea that the CPR on Proving *does not* measure a student's overall creativity or evaluate the validity of a final proof. Rather, the emphasis is on the proving process and the development of practices that mathematicians report can lead to creative thinking, all while students develop awareness of their metacognitive processes.

We documented possible use of the rubric in class through our pilot studies.

Students of these proof-based courses received copies of the rubric. The instructors walked the classes through rubric's by applying it to a student's work. A whole-class discussion helped students understand the language of the rubric, the importance of looking at scratch work to explore the development of ideas and reasoning, and the possible ways of using it to improve the creative process of proof construction.

The CPR on Proving can be used with any proof task. However, it may yield more information for students and instructors when used on tasks that elicit exploration or when the key idea(s) of the proof are not directly seen. When such tasks are assigned for an in-class activity, students can use the rubric to reflect on their or their peers' proof production by making the categories the basis for discussion.

In addition, an instructor may include an assignment to use the rubric on a sample proving process.

Figure 1. The Creativity-in-Progress Rubric (CPR) on Proving.

<b>MAKING CONNECTIONS:</b>	<b>Beginning</b>	<b>Developing</b>	<b>Advancing</b>
Between Definitions/Theorems	Recognizes some relevant definitions/theorems from the course or textbook with no attempts to connect them in their proving	Recognizes some relevant definitions/theorems from the course and attempts to connect them in their proving	Implements relevant definitions/theorems from the course and/or other resources outside the course in their proving
Between Representations <sup>1</sup>	Provides a representation with no attempts to connect it to another representation	Provides multiple representations and recognizes connections between representations	Provides multiple representations and uses connections between different representations
Between Examples	Generates one or two specific examples with no attempt to connect them	Generates one or two specific examples and recognizes a connection between them	Generates several specific examples and uses the key idea synthesized from their generation

<sup>1</sup> We define a *mathematical representation* similar to NCTM's (2000) definition. It includes written work in the form of diagrams, graphical displays, and symbolic expressions. We also include linguistic expressions as a form of lexical or oral representation. For example, a student can use the lexical or oral representation, "the intersection of sets  $A$  and  $B$ "; a Venn Diagram to depict his/her mathematical thinking; a symbolic representation  $A \cap B$ ; or set notation  $\{x|x \in A \text{ and } x \in B\}$  (which is also a symbolic representation). Note the last two representations are in the same category, e.g. symbolic, but they are still considered two different representations.

<b>TAKING RISKS:</b>	<b>Beginning</b>	<b>Developing</b>	<b>Advancing</b>
Tools and Tricks <sup>2</sup>	Uses a tool or trick that is algorithmic or conventional for the course or the student	Uses a tool or trick that is model-based or partly unconventional <sup>3</sup> for the course or the student	Creates a tool or trick that is unconventional for the course or the student
Flexibility <sup>4</sup>	Begins a proof attempt (or more than one proof attempt), but uses only one approach	Acknowledges and/or uses more than one proving approach, but only draws on one proof technique	Uses more than one proof technique
Posing Questions	Recognizes there should be a question asked, but does not pose a question <sup>5</sup>	Poses questions clarifying a statement of a definition or theorem	Poses questions about reasoning within a proof
Evaluation of Proof Attempt	Examines surface-level <sup>6</sup> features of a proof attempt	Examines an entire proof attempt for logical or structural flow	Examines and <i>revises</i> an entire proof attempt for logical or structural flow

<sup>2</sup> Based on the Originality category from Leikin (2009).

<sup>3</sup> Learned in a different context.

<sup>4</sup> A proof attempt is a continuous, sustained line of reasoning focused on a single theorem or conjecture. A proof approach is a proof attempt in which a new or different (to the prover) idea is introduced. Finally, a proof technique is a proof approach that addresses the overall logical structure of the proof. Common proof techniques include induction, proof by cases, direct proof, contradiction, and contrapositive.

<sup>5</sup> For example, a student writes a "?" next to something.

<sup>6</sup> Surface-level features include technical, computational, and line-to-line logical details.

This affords students an opportunity to reflect on their own proving process while giving feedback to others. The instructor could use students' self-reflections to individually guide the students on their development of proof construction.

Additionally, students in our pilot studies reported that they used the rubric as a checklist, often referring to it when they felt stuck. Impasses are an inevitable aspect of proving, and having a tool to assist or guide someone through the struggle may be beneficial.

The mathematical creativity may not lie in the student's final proof, but rather in the winding path that the

student's work takes. Many students can be incredibly creative and create invalid proofs. CPR on Proving might be the tool that can help these students to recognize their creative potential as well as guide them to create valid proofs on this path. We invite others to use the CPR in their courses and provide feedback. 

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