How teaching to foster mathematical creativity may impact student self-efficacy for proving

Mathematical creativity has been emphasized as an essential part of mathematics, yet little research has been done to study the effects of fostering creativity in the tertiary mathematics classroom. In this paper, we explore how fostering mathematical creativity may impact student self-efficacy for proving. For this, we developed new methods to study evidence of instructor use of Sriraman's (2005) five principles for fostering mathematical creativity and changes in student self-efficacy via Bandura's (1997) four sources of self-efficacy. This revealed associations between four of the five principles and changes in student self-efficacy for proving, along with two instances where the combined use of principles may have provided students greater opportunities for building self-efficacy for proving. The implications of these results for teaching and future research are discussed.

Keywords: mathematical creativity, discrete math, proving, self-efficacy

For over a century mathematics and mathematics education researchers have endeavored to better understand the role of creativity in mathematical thinking and problem solving (Poincare, 1946; Mann, 2006; Haavold, 2016). Since mathematics education and research are both ripe with the potential for creativity, a research stream has developed offering various ways to foster creativity in mathematics (Zaslavsky, 1995; Mason, 2005; Leikin, 2007; Shriki, 2008; Holton, 2009; Sriraman, 2005; Leikin, 2014; Savic, 2017a). At the same time, there is little research studying the impact of fostering mathematical creativity on individual cognitive constructs such as self-efficacy (Mathisen & Bronnick, 2009).

This paper focuses on studying the impact of fostering creativity on student self-efficacy in mathematics due to the powerful role self-efficacy plays in mediating student achievement (Pajares & Kranzler, 1995; Randhawa, Beamer, & Lundberg, 1993). In a variety of contexts, people with high self-efficacy have been shown to experience increased motivation, engagement, and resilience to adversity (Bandura, 1997); demonstrate increased use of strategic thinking; manage their time better; are more persistent; and are less likely to reject correct solutions (Bouffard-Bouchard, 1990, Bouffard-Bouchard et al., 1991). Furthermore, self-efficacy has been identified as a better predictor of mathematical performance than prior ability or experience within mathematics (Siegel, Galassi, & Ware, 1985; Pajares & Kranzler, 1994).

Tan, Li, and Rotgans (2011) demonstrated several connections between secondary students' experience in the classroom and domain-general *creative self-efficacy* but called for more research designs to explore how various classroom factors impact self-efficacy. We add to this literature by studying *self-efficacy for mathematical proving*, a little-studied construct in mathematics education research (Iannone & Inglis, 2010). This serves to highlight the ways in which fostering creativity in tertiary mathematics education may influence student self-efficacy for mathematics.

The purpose of this article is two-fold. Firstly, we introduce a theoretically-based methodology for both quantitatively and qualitatively studying the effect of creativity-fostering instruction on student self-efficacy for proving. Secondly, we study how the sources of self-efficacy afforded by the use of creativity-fostering instruction can serve to build student self-efficacy for proving. In viewing student self-efficacy for mathematical proving as a domain-specific creative trait (Baer, 1998), this research also sheds light on what classroom environments may best foster creative capacities of mathematics students at the tertiary level.

1. Theoretical Background

1.1 Mathematical Creativity in the Classroom

From Leikin's (2009) and Silver's (1997) use of Torrance's (1966) constructs of creativity, to use of Wallas' four stages of creativity (1926) in pure mathematics (Hadamard, 1945), mathematical creativity has often been measured or described in relation to the individual (either the student or mathematician). At the same time, there is considerable variation in the ways individual mathematical creativity has been defined (Mann, 2006). While recognizing the multi-faceted nature of creativity (Moore-Russo & Demler, 2018), for the purpose of this study, we define *mathematical creativity* as one's process of offering new solutions or insights that are new or unexpected for the student with respect to their mathematical background. This definition is based on Savic et al. (2017b), influenced by the perspectives of Liljedahl and Sriraman (2006). One can further categorize this definition as relative to the individual (Beghetto & Kaufman, 2007), process-oriented (Pelczer & Rodriguez, 2011), and domain-specific (Baer, 1998) mathematical creativity. While this definition is appropriate for studying actions for fostering students' mathematical creativity, it is methodologically difficult to measure the extent to which a student's actions or behavior are creative, i.e. new or unexpected to the creator. Thus, in this study, rather than measure what is creative, we leave that judgment to the student, looking instead at their self-efficacy for proving (section 1.2).

To better understand the role of mathematical creativity in the classroom, we were interested in studying observable actions[1] instructors use to foster creativity in the mathematics classroom. Zazkis and Holton (2009) provided an overview of a range of ways instructors can foster creativity at tertiary mathematics, citing Zaslavsky's (1995) work on *open-ended problems*, Shriki's (2008) work on students creating new definitions, Leikin's (2007) *multiple-solution tasks*, and Watson and Mason's (2005) *learner-generated examples*. Zazkis and Holton (2009) further detailed their own tasks and classroom actions, including starting a graph theory course with an open-ended question about edges and vertices, or understanding π by asking students to calculate the ratio of perimeter to the "segment connecting the 'centre' to the 'corner'" (p. 361) of squares and triangles. More recently, Leikin (2014) studied how *multiple-solution tasks* were used in a university course for prospective teachers, observing discussions in class that focused on the multiple geometric proofs constructed by the students and commenting on the misnomer of tasks requiring a single solution.

In researching mathematical creativity in the K-12 classroom, Sriraman (2005) conjectured five principles that can be "applied in the everyday classroom to maximize the potential for creativity in the classroom" (p. 26). These principles, described below, are derived from the mathematical creativity literature along with mathematicians' experiences of creating and publishing their results. While these principles (nor the previous literature described above) are

[1] Including non-actions, i.e. not acting or speaking a prescribed way in a given situation; for example, not answering a student's question, "Is this correct?"

by no means exhaustive of the ways in which an instructor can foster mathematical creativity in the classroom, they provide a general framework used in this study for identifying a range of instructor actions that can be viewed as creativity-fostering.

1.1.1. The Gestalt Principle. The Gestalt principle involves instructors providing students opportunities in or out of class to "engage in suitably challenging problems over a protracted time period, thereby creating the opportunities for the discovery of an insight and to experience the euphoria of the 'Aha!' moment" (p. 26). The "Aha" moment is the crucial third step in Wallas' (1926) four-step creative process (preparation, incubation, illumination, and verification) due to the chance for a solution.

1.1.2. The Aesthetic Principle. Sriraman stated that "mathematicians have often reported the aesthetic appeal of creating a 'beautiful' theorem" (p. 27). The aesthetic principle applies to instructors valuing solutions that are efficient or elegant, utilize unusual proving techniques, or come from diverse topics of mathematics, or make. The teacher explicitly promotes the aesthetics of students' mathematical processes or products. They may also discuss the beauty in the history of mathematics.

1.1.3 The Free-Market Principle. By Sriraman's recommendation, "teachers should encourage students to take risks" (2005, p. 28). The free market principle involves creating a classroom environment that allows students to freely input ideas, thoughts, and solutions. This also includes teachers encouraging students to "defend their ideas upon scrutiny from their peers."

1.1.4 The Scholarly Principle The scholarly principle involves creating a classroom environment "in which students are encouraged to debate and question the validity of… approaches to problems…, be encouraged to generalize the problem and/or the solution, as well as pose a class of analogous problems" (p. 28). This principle is equated to fostering students' role as scholars and allowing them to build mathematics off one another.

1.1.5 The Uncertainty Principle. The uncertainty principle "requires that students be exposed to the uncertainty and the difficulty of creating mathematics" (p. 28). According to Sriraman's, the teacher should take actions "cultivating this trait of perseverance" (p. 28). This requires that instructors attend to frustrations and balance difficulty while also developing a tolerance for ambiguity.

Recently, these five principles have been utilized as a lens for understanding how mathematics instructors view fostering creativity in the classroom (Moore-Russo & Demler, 2018), as well as in developing a framework for studying student attitudes and experiences of creativity (Haavold, 2016). Yet, a detailed characterization of actual classroom implementation of the five principles, to our knowledge, has not been offered. Thus, the results of this study (section 3) serve to further ground and expand the theory behind the five principle, as well as extend it to the tertiary level. Based on these results, we were interested in studying how instructor-use of these five principles may impact student experience and development in mathematics, specifically in relation to student *self-efficacy for proving*, a construct not yet examined in the creativity literature.

1.2 Self-efficacy for proving

Bandura (1997) defined perceived self-efficacy as "beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments" (pg. 3) and is a central construct of social cognitive theory. According to social cognitive theory, self-efficacy is domain-specific: one's self-efficacy will vary depending both the task in question and context one is working in (Bandura, 1997). Therefore, we define *self-efficacy for proving* as one's beliefs in their own capabilities to organize and execute the actions required to produce justifiable mathematical proofs.

In this paper, we view *proof* at the university introductory-level as a logical justification of a mathematical statement (Weber, 2005). However, we consider this definition within a broader view of proof as "as a series of ideas and insights rather than [just] a sequence of formal steps" (Hanna & de Villiers, 2012). Therefore, *proving* refers to the process of justifying a mathematical statement or conjecture (Hsieh, Horng, & Shy, 2012). Previous research on proving at the tertiary level has focused on students' cognition, including logical skills necessary for proving (Selden and Selden, 1995), types of reasoning and problem-solving processes used by students (Weber, 2005), and proof schemes (Harel & Sowder, 1998) or "arguments [one] uses to convince [themselves] and others of the truth or falseness of a mathematical statement" (Housman & Porter, 2003, p. 140).

There is less research studying the impact of affective aspects of students' experience, such as self-efficacy, in proving (Iannone & Inglis, 2010). Selden, McKee, & Selden (2010) studied affect in relation to the behavioral schemas used by students in proving. Iannone & Inglis (2010) studied university students' self-efficacy for proving and their proving abilities, finding a positive correlation. However, their study does not explain any cause for such a correlation. Selden and Selden (2010) identified the importance of self-efficacy in supporting students' three useful actions in proving: exploring, reworking an argument in the case of a suspected error or wrong, and validating a completed proof. They hypothesized that few students attempt these actions due to a lack of self-efficacy.

Furthermore, Selden and Selden (2013) hypothesized that self-efficacy is an important part of "much of creative cognition in general" (p. 4). In fact, one early model of creativity, the componential model of creativity (Amabile, 1983), described both domain-relevant skills (including knowledge about the domain) as well as creativity-relevant skills (i.e. exploring new cognitive pathways, suspending judgment) as components contributing to creative performance. The componential model of creativity has been widely used to study domain-general creative self-efficacy (Tan, Ho, and Young, 2007; Tan, Li, and Rotgans, 2011; Sangsuka & Siriparp, 2015). However, according to Beghetto and Karworski (2017), there is a need for more robust, domain-specific measurements of creative self-efficacy. Thus, we add to the literature by studying self-efficacy for proving as a domain- and task-specific measurement of one's self-efficacy for a potentially creative task, mathematical proving.

In this study, we are interested in better understanding *how* students can gain self-efficacy for proving through classroom experience. Bandura (1997) outlined four primary sources of self-efficacy information: enactive experiences, vicarious role-modeling, verbal persuasions, and physiological reactions, described below.

1.2.1. Enactive Experiences. Enactive experiences refer to one's own successes in accomplishing a given task. An example of an enactive experience could be one's experience of solving a difficult proof and successfully explaining it to someone else. According to Bandura (p. 8), enactive experiences are often the most powerful source of self-efficacy since one's own experiences provide a reliable indication of future ability. It is of note that these sources do not directly influence self-efficacy; it is through cognitive processing and reflective thought that these sources of information are "selected, weighted, and integrated into students' self-efficacy judgements" (p. 79). However, any change in one's self-efficacy operates through one or more of these four sources.

1.2.2. Vicarious Role-modeling. Vicarious role-modeling involves observation of someone else's competencies through which, by self-comparison, the observer bases judgments of their own ability. Observing someone else present their own proof could provide some indication of the observer's own proving ability, to the degree that the observer identifies, or sees themselves similar to, the presenter. The observer could gain self-efficacy from that observation, and perhaps be more confident approaching a similar or future problem.

1.2.3. Verbal Persuasion. Verbal persuasion involves direct verbal appraisal of one's ability by someone else. Telling a student, "I believe you have the resources to prove this" can serve as some indication of ability but depends on the credibility of the persuader and the degree to which such "positive appraisal is within realistic bounds" (p. 101). Verbal persuasion is usually considered less reliable than previous two sources, since it conveys beliefs that are described rather than observed.

1.2.4. Physiological Reactions. Physiological reactions can include feelings of strength and stamina, or physical or emotional stress or fatigue. Feeling well rested or comfortable in the classroom are indicators of ability, while feeling of stress or fatigue are "signs of vulnerability to dysfunction" (p. 106).

1.3 Research Question

Drawing on the above characterization of the five principles for fostering creativity and the four sources of self-efficacy, our two research questions are:

1. What methodologies are appropriate for measuring creativity-fostering instruction in the classroom and changes in student self-efficacy for proving, specifically in an introduction-to-proofs course?
2. How may teaching actions for fostering creativity (categorized by Sriraman's five principles) impact how students gain or lose self-efficacy for proving in an introduction-to-proofs course?

2. Methods

The data for this study was collected in a Discrete Mathematic course at a research-intensive university in the central USA. This course utilized inquiry-based learning, defined by the Academy of Inquiry-Based Learning as: "a form of active learning in which students are given a carefully scaffolded sequence of mathematical tasks and are asked to solve and make sense of them, working individually or in groups" (Academy of Inquiry-Based Learning, n.d.). In this course, classroom observations (section 2.1), online surveys (2.2), and student interviews (2.3) were all collected and compared to explore the relationship between presence of the Five principles in the classroom and changes in student self-efficacy. This served to: (1) corroborate evidence of the use of the five principles in the classroom and (2) document changes in students' self-efficacy.

We piloted these methods in an 8-week summer session. After analyzing pilot data and making changes, primary data was collected in the fall semester of 2017, taught by Dr. F. Each class was videotaped, 23 students took a beginning-of-semester online survey, 21 students took an end-of-semester online survey, and 4 students were interviewed.

2.1 Online Surveys

Two online survey instruments were designed to measure student experience of the five principles and their self-efficacy for proving. Named the Five Principles Survey (5PS), it consisted of ten questions, two per principle, collecting students' experience of each principle (see Appendix A). The questions were randomized for each student. The 5PS was given at the end of the semester (Survey 3), rating their experience in Discrete Mathematics that semester.

The Self-efficacy for Proving Scale (SEPS) consisted of three specific proving statements in which students were asked to rate their confidence of five subtasks related to proving each statement. The researchers closely followed Bandura's recommendations for constructing self-efficacy scales (2006), as well as Beghetto and Karworski's (2017) suggestions for using and refining domain-specific measurements of creative self-efficacy. This is distinct from the common practice of using general statements of mathematical ability or experience to measure "self-efficacy" (Iannone and Inglis, 2010).

The three proving subtasks served to orient students to the domain of interest (mathematical proving at the undergraduate level), each designed to be accessible to students with no prior experience with formal proof. Hammack's (2013) proof textbook served as the main source for these problems. To gauge gradation of challenge of the tasks, both researchers characterized each statement on the continuum of problem difficulty offered by Selden & Selden (2013, pp. 303-305), shown in Table 1. For each of the three surveys, the first author selected one proving statement of each difficulty type, ensuring that these statements were not included in class or in any assigned homework, quiz, or exam.

For each proving statement, to obtain a measure of student's abilities related to the *process* of proving and to provide context for students potentially unfamiliar with formal proving, students were asked rate their confidence in their ability (on a scale of 0% confidence to 100%) to do the following five subtasks related to proving:

1. Understand and informally explain why a statement is true or false.
2. Explore new ideas to come up with ways to start your proof.
3. Use various representations (numbers, pictures, tables, words) to structure your thinking
4. Formally write out and justify each step of your proof.
5. Examine your proof for accuracy and identify any missing steps.

These subtasks were derived from Hsieh, Horng, and Shy's (2012) Exploration-Proving spectrum (EP-spectrum) for proving, which centers on the concept of justification and, at the same time, considers *proof in the classroom* more broadly "as the product of a spectrum of activities starting with exploration, and progressing to the stages of conjecturing, informal explanation, and justification" (pg. 288). This both aligned with our perspective of proving (section 1.2) and provided a more detailed characterization of the courses of actions required to produce justifiable proofs.

Three versions of this survey, each with distinct proving statements, were given, at the beginning (Survey 1), middle (Survey 2), and end (Survey 3) of the semester. All nine proving statements are included in section 3.2. The SEPS use in survey 3 is provided in Appendix B.

Table 1: Selden and Selden's (2013) continuum of problem difficulty

Type	Description	Example Statement from Survey
1 – Very-routine	Can easily be proven from previous results	Prove or disprove: If n is an odd integer, then $n^4 - n$ is even.
2 – Moderately-routine	Requires formulating and proving a lemma (or trick) that is relatively easy to notice, formulate, and prove	Prove or disprove: The inequality $2^x \geq x + 1$ is true for every positive real number x.
3 – Non-routine	Requires formulating and proving a lemma (or trick) that is hard to notice, formulate, and prove	Prove or disprove: There does not exist a real number x for which $x^4 < x < x^2$.

2.2 Classroom Observations

One class session from the fall semester was randomly chosen from the beginning (first five weeks), middle (weeks 6-10), and end (weeks 11-15) of the fall semester. Each session was viewed by both researchers and coded for both explicit and implicit evidence of instructor use of each of the five principles of creativity. We defined *explicit evidence* of the principles to be overt verbal instruction aligned with one or more principle, and we defined *implicit evidence* to be situations or interactions from which the instructor's influence for one or more principles could be inferred. Since we chose the theoretical framing of the five principles prior to coding, this is similar to the provisional coding described by Saldaña (2009) in his fundamental qualitative research manual. Differences in codes were discussed until arriving at an agreement for each coded action. The researchers chose not to calculate Cohen's Kappa for interrater reliability and instead "rel[ied] on intensive group discussion and simple group 'consensus' as an agreement goal" (Harry, Sturges, & Klingner, 2005, p. 6; as cited by Saldaña, 2009, p. 28). This data collection method served mainly to provide direct evidence of which principles, if any, were used in the classroom.

2.3 Student Interviews

Four students (with pseudonyms Fannie, Fred, Frank, and Francisca) from the Fall semester (taught by Dr. F) participated in a post-semester interview. Each student was asked questions about their classroom experience, their relative confidence in proving now, and how they gained confidence in their class. A full list of the questions is provided in Appendix C. Additionally, the students from the fall semester were given 30 minutes at the beginning of the interview to prove the same three statements used in the end-of-semester self-efficacy survey. Then they were asked to describe their proving process and indicate whether they had previously proved seen a proof of these statements. This was to further validate the self-efficacy survey.

Each interview was transcribed, removing words such as "like" and "um" and "and stuff" for readability. Each interview was coded, once for explicit or implicit evidence of the instructor's use of Sriraman's (2005) five principles for maximizing creativity, and again for evidence of Bandura's (1997) four sources of self-efficacy. For coding the student interviews, we defined *explicit evidence* of the principles to be as overt teacher actions aligned with one or more principle described by student. This frequently included direct mention of the instructor. We defined *implicit evidence* to be situations cited by the students from which instructor influence for one or more principles could be inferred. Sources of self-efficacy were coded only when a potential source (enactive experiences, vicarious role-modeling, verbal persuasion, physiological reaction) was described *in relation to* a change in student confidence in proving. Codes were compared, and any discrepancies were discussed until a consensus was reached. Intersections between the five principles and self-efficacy codes were then analyzed. In our analysis, we used the general term *association* to discuss such intersections between one or more of the five principles and source of self-efficacy. This follows the general definition of an association as "a connection between ideas or things," distinct from the statistical use of the term. However, we note that in our analysis an association was drawn only when the language used by the student describes causality between the coded constructs.

To illustrate our coding, consider the following student response to the question, "What do you think contributed to your gaining confidence in proving?":

> *Fred*: When we would do the peer discussions in class, I would see how somebody else did it, and then I would be like "okay that makes a lot of sense, like how you, kind of played around with, and how you got to where you went." And usually after class, I would have a break for six hours in between my next class. So, I would, a lot of times, go back and I can redo the whole, like two homework problems.

This was coded for explicit use of the scholarly principle due to the instructor's use of peer discussion in class, specifically where students (in this case, Fred) built off and evaluated one another's ideas (in this case, "somebody else"). This response was also coded for gaining self-efficacy via vicarious role-modeling, since the Fred attributed working with his peers and observing their proving process as contributing to his gaining confidence for proving. Thus, an association between the scholarly principles and gaining self-efficacy via vicarious role-modeling could be inferred from this response.

3. Results

To begin to answer our research questions, we first describe the evidence of instructor enactment of the five principles from online surveys, classroom observations, and student interviews (3.1). Then, we discuss the evidence from online surveys and student interviews of a pre/post-semester change in students' self-efficacy (3.2). Finally, having observed consistent the enactment of the five principles in class as well as a change in student self-efficacy, we present evidence from the student interviews of possible associations between the five principles and the ways students gained (or lost) self-efficacy for proving (3.3).

3.1 Evidence of Instructor Enactment of the Five Principles

Evidence of instructor use of all five principles were found in all three data groups. While the online surveys show common use of the five principles, we further investigated the classroom observations and student interviews in order to demonstrate concrete actions of teaching aligned with Sriraman's five principles.

3.1.1 Online Surveys Figure 1 shows a histogram of students' responses from the Five Principle Survey (5PS) given at the end of the semester. The median response for question 2, "How often did you experience the joy of arriving at a solution after working on a problem or proof for several days?" (Gestalt) was "6-10 times per semester." Question 7 (scholarly) and question 9 (uncertainty) had the same median responses. The remaining questions had a median response of "weekly."

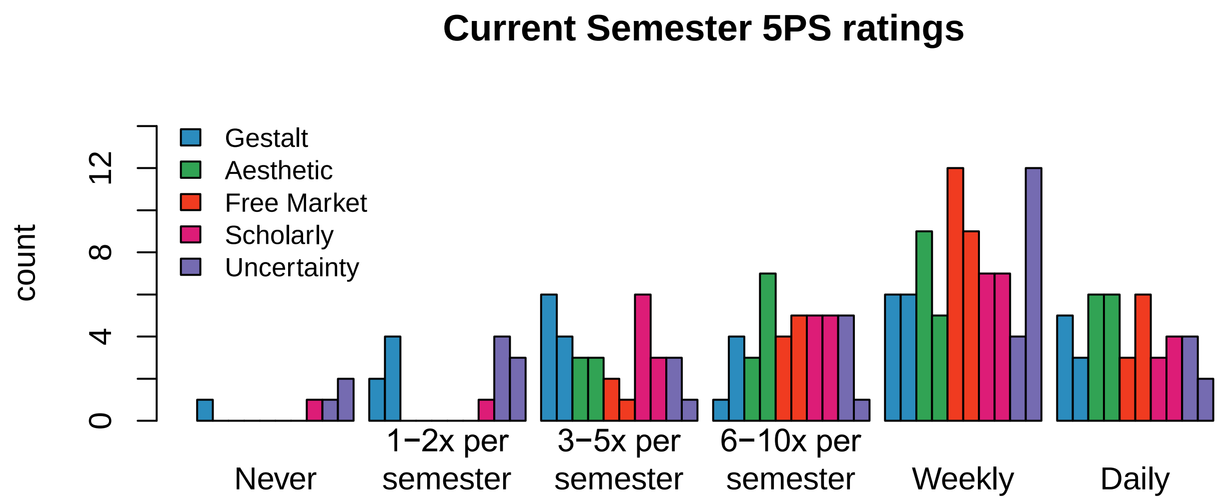

Figure 1: Student end-of-semester Five Principle Survey Ratings

This gave evidence that 79-100% of student responses described experiencing a given principle at least 3 times during the semester; 63-95% of student responses described experiencing a given principle at least 6 during the semester; and 42-74% of student responses described experiencing a given principle weekly or daily (see table 2).

Table 2: Cumulative Frequency of Student Rating for 5PS

Frequency of student experience of the five principles (5PS)	Gestalt		Aesthetic		Free Market		Scholarly		Uncertainty	
	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
at least once	95%	100%	100%	100%	100%	100%	100%	100%	95%	89%
at least 3-5 times/sem	89%	79%	100%	100%	100%	100%	100%	95%	79%	79%
at least 6-10 times/sem	63%	63%	84%	84%	89%	95%	68%	79%	68%	74%
at least weekly	58%	42%	68%	53%	74%	68%	47%	53%	42%	68%
daily	26%	16%	26%	26%	16%	26%	16%	16%	21%	11%

3.1.2 Classroom Observations In the first of the three class periods that were coded, all five principles were observed. At the beginning of class, Dr. F and the teaching assistant (TA) discussed how they were giving students an opportunity to redo some of the homework problems. The TA had given feedback to some of these problems which had been turned in online through a learning management system by providing open-ended responses to students' questions about their work, where they got stuck, etc. Several of these problems had been discussed the prior class period. Dr. F said that "we are giving you all a chance to redo some things, mainly because it is for you, not for us." This was coded for the explicit use of the Gestalt Principle for allowing students freedom of time and movement to foster "aha" experiences.

After this, Dr. F began responding to a student question about "if-then" statements, asking, "'If x is an element of A, then x is an element of B.' Which [set theoretic] statement is that?" Students variously responded, "A union B," "A intersect B," and "A subset B." Without responding to the accuracy of any student responses, Dr. F said, "go to your notes." As students began agreeing "It's 'A subset B,'" Dr. F asked, "what is another truthful statement about this?" One student responded, "A equals B." Several students disagreed, which point Dr. F said, "Hold on, hold on, hold on. Why do you say, 'A equals B'?" This student commented, "Because if A is bigger than B, it's not contained in B. But if it's the same size as B, then they're equal," to which Dr. F responded, "In fact, 'A equals B' can be one case. There [are] many cases, and one of them is 'A equals B.' What about some others with elements?"

The above interaction was coded for implicit uses of the uncertainty, scholarly, and free market principles. The statement, "Go to your notes," was coded for use the uncertainty principle since the instructor, in not responding the accuracy of students' response, implicitly used students not knowing as an opportunity for them to be comfortable in finding their own answers to uncertainty. The instructor allowing students to debate with one-another in this interaction was coded for use of the scholarly, since this allowance helped engage students in challenging the validity of their own responses. Finally, we coded the engagement of a solution that was not fully correct ("In fact, 'A equals B' can be one case") for implicit use of the free market principle since engaging a potentially wrong solution and using it to explain how their thoughts fit into the bigger picture may have served to foster future risk taking more than focusing on the correctness of the students' response.

This led to a conversation of contrapositive, inverse, and converse statements. At one point, Dr. F incorrectly stated, "this statement is the inverse," to which a student corrected "[it's the] converse, I just looked it up." This was coded for implicit use of the scholarly principle, since this illustrated a norm running throughout the course – students had access to all the notes, were repeatedly encouraged to refer to the notes, and were challenging mathematical authority to construct their own understanding from the notes.

Toward the end of this discussion, Dr. F explained, "I believe this statement [if, then] is why we learn math. I believe that calculus, adding fractions, boils down to being logical. I believe this is why for 3500 years we have been learning math: to be more precise when we speak and talk." This was coded for explicit use of the aesthetic principle. Dr. F was conveying the beauty, elegance, and precision of mathematical communication. In the remainder of this class period, we coded one or more instances of each principle.

In the second class, six instances of use of the free market and scholarly principles were coded, several of which came about as a result of the instructor offering Fannie the opportunity to lead the class discussion. This action was coded for explicit use of free market principles for his encouraging Fannie to take this risk. Fannie's response to this was, "oh this is so exciting," to which she proceeded to lead class discussion (with minor guidance from Dr. F) the remainder of the class. Two instances of scholarly principle were coded in the interaction with Fannie and the class, as well as another instance of the free market principle, for Dr. F creating a safe environment for which students could engage in this discussion.

In the third class, two instances of explicit use the free market principle were coded in relation to a course requirement (worth 5% of their total grade) called "Productive Failure." This requirement involved students presenting an experience where they failed in their proving process and explaining how it proved productive in the end. In response to a student question, "How do I encourage my friend to present a productive failure?", Dr. F said, "I make mistakes in lecture, and am getting better at talking about my own failures. Having a difficulty and talking to someone else about it relieves the weight." The counted instances of the coding for the five principles for all three class periods are shown in Table 3.

Table 3: Frequency of coding of three randomly-selected class periods

Principle	Beginning	Middle	End
Gestalt	2	1	1
Aesthetic	3	1	0
Free Market	3	6	3
Scholarly	3	6	2
Uncertainty	2	3	6

3.1.3. Student Interviews The analysis of the student interviews illustrated a range of cases in which the students cited use of the five principles in the classroom. Table 4 summarizes frequency for which each principle was coded in the interview. For example, Francisca stated that she experienced "inherent curiosity" from the course because

> [Dr. F] would be like "so why did you do that?" And at first it was just like "I don't know cuz that's what you do." And it was like "no, why did you do that." And like that constantly asking over and over -- "well why are you doing the thing that did?" -- is the reason why.

This was coded as an uncertainty principle, since the instructor utilized questioning rather than attending to correctness in the classroom. More examples of Dr. F's utilization of the five principles through the student interviews are provided in Section 3.3.

Table 4: Frequency of coding of the five principles for each interview

Principle	**Fred**	**Fannie**	**Frank**	**Francisca**
Gestalt	5	3	1	4
Uncertainty	4	1	4	5
Aesthetic	0	0	1	0
Scholarly	4	2	1	5
Free Market	1	2	1	1

3.2 Evidence of Change in Student Self-efficacy

In this subsection, we provide the evidence of changes in student self-efficacy for proving in the online surveys and student interviews. Comparison of the beginning-of-semester and end-of-semester online surveys provide evidence that 16 out of 19 students gained self-efficacy for proving during the course. From student interviews, the results of the task-based component and their corresponding self-efficacy scores were compared. Finally, we review the evidence from the interviews of the sources of self-efficacy that were experienced by these students. Here, a potential source of self-efficacy was identified when a student acknowledged a change in their own self-efficacy in relation to one or more of the four sources of self-efficacy.

3.2.1. Online Surveys For comparison of beginning, middle, and end-of-semester the SEPS ratings, each students' self-efficacy rating for a given proving statement was calculated for as the mean of the five subtask ratings for that statement. The mean and standard deviation of these ratings are shown Table 5. Then, each student's self-efficacy rating was calculated as the mean of the three proving statement ratings. As an example, on Survey 3, Fannie rated her self-efficacy for five subtasks (see section 2.2) on the first proof statement were 100, 90, 80, 90, and 90, respectively, giving an overall self-efficacy rating of for the first proof statement of 90. Her self-efficacy for the remaining two proof statements were 82 and 90, giving an overall SEPS rating of 88.

Table 5: Mean and standard deviation of self-efficacy scores for each proof statement used

Survey	Level	Statement	Mean	SD
1 (n=23)	1	If n is an odd integer, then $n^2 + 1$ is even.	64.09	15.64
	2	If a, b, c are positive integers, and ab, bc, and ac all have the same parity (are all even or all odd), then a, b, and c all have the same parity.	52.09	24.10
	3	If a and b are integers, then $a^2 - 4b \neq 2$.	57.57	21.88
2 (n=24)	1	If n is an odd integer, then $n^4 - n$ is even.	76.00	12.50
	2	The inequality $2^x \geq x+1$ is true for all positive real numbers x.	74.17	14.36
	3	There does not exist a real number x for which $x^4 < x < x^2$.	73.33	14.90
3 (n=21)	1	If $x, y \in \mathbf{R}$, then $\lvert x + y\rvert \leq \lvert x\rvert + \lvert y\rvert$.	84.67	10.13
	2	If n is an integer, then $1 + (-1)^n(2n - 1)$ is a multiple of 4	81.43	13.22
	3	Every odd integer is the difference of two squares.	77.81	16.50

The changes in student experience of the five principles and their self-efficacy can be seen in Figure 2, which plots the students end-of-semester vs. beginning-of-semester SEPS scores. The data from Fannie, Fred, and Frank are highlighted in square, triangle, and diamond respectively. We did not get a response from for Francisca on the final survey. The remaining un-interviewed students (16 out of n=19) are shown in grey. As an example of one data points, we describe Fannie's data (the square data point). Her beginning-of-semester self-efficacy score was 42, plotted on the horizontal axis, and her end-of-semester self-efficacy scores was 88. The change in her self-efficacy rating, 46, is represented by the vertical distance from the square to the line y=x.

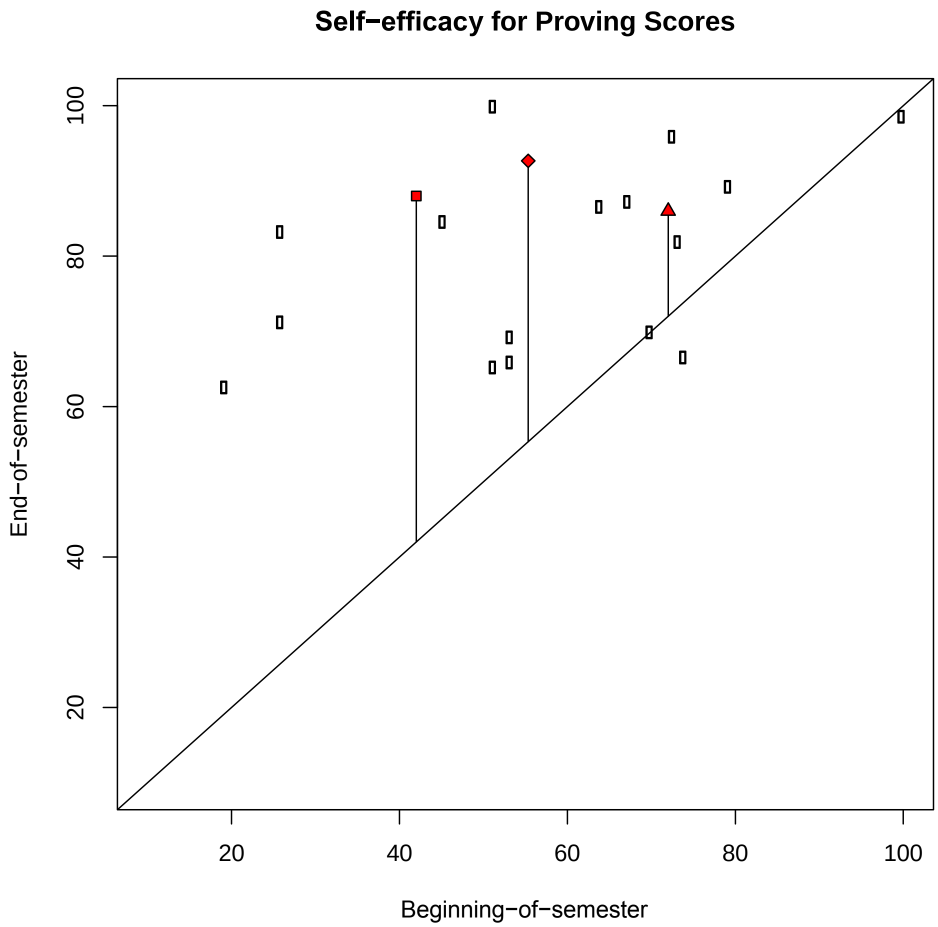

Figure 2: Student end-of-semester vs. beginning-of-semester SEPS scores.

Several tests of the statistical significance of the changes in self-efficacy were conducted. Firstly, a one-tailed paired Wilcoxon signed ranks test was conducted ($p=1.907 \times 10^{-5}$) giving evidence that the frequency distribution of end-of-semester SEPS scores are shifted to the right of the distribution of the beginning-of-semester scores. Next viewing the SEPS scores as a continuous variable (as the mean of 15, 11-point scales), we verified that the beginning- and end-of semester scores did not violate assumptions of normality. Then a paired two sample T-test was conducted showing a significant ($p=3.626 \times 10^{-5}$) change in student self-efficacy ratings, with a 95% confidence interval for change in SEPS ratings of (14.54, 32.83).

3.2.2 Student Interviews Three of the four students interviewed took Survey 3 prior to their interview. Thus, we can also use the task-based portion of the student interviews to analyze the degree to which these students' self-efficacy ratings (for the three proving statements on survey 3) reflected their proving abilities demonstrated on the actual tasks. Fannie rated her self-efficacy for proving the three statements as 90, 82, and 92 respectively, and Frank rated his self-efficacy for proving the three statements as 94, 94, and 90 respectively. Both Fannie and Frank proceeded to successfully prove all three statements. Fred rated his self-efficacy for proving the three statements as 84, 88, and 86 respectively, and proceeded to successfully solve the first statement. Fred did not finish proving the remaining statement, making several proof attempts, including a proof by induction on second, and an attempt at a proof by contradiction on the third. Frank was the only student who described having seen any of these proofs before (the first statement, the triangle inequality).

The coding of the student interviews provided evidence that the four students interviewed experienced changes in self-efficacy via enactive experiences and vicarious role-modeling. Only one instance of verbal persuasion serving as a source of self-efficacy was coded, and no cases of physiological reactions serving as a source of self-efficacy were coded. The coding for each sources of self-efficacy are shown in Table 6. The frequency of coding for positive sources of self-efficacy are listed in each cell, along with the frequency of coding for negative sources of self-efficacy in parenthesis.

Table 6: Frequency of coding the sources of self-efficacy

Source of Self-efficacy	**Fred**	**Fannie**	**Frank**	**Francisca**
Enactive Experiences	3 (2)	6	3	6 (2)
Vicarious Role-Modeling	3 (1)	4	2	2
Verbal Persuasion	0	0	0	1
Physiological Reactions	0	0	0	0

Below are examples of coding for both positive and negative sources of self-efficacy. When asked what helped her gain confidence for proving, Fannie responded:

> I think that having us do it on our own pretty much all the time was key. Because I don't think I would get the same, I don't think I would have had the same understanding of it if it was just presented to me rather than me figuring it out for myself, if that makes sense.

This was coded for evidence of enactive experiences (doing it "on our own pretty much all the time") contributing to her gaining confidence. Compare this with Fred's response to the question: "where did you struggle and what prevented you from gaining confidence?"

> I'd say like the beginning a semester… It took me till the middle of September to really devote a lot of time to this class, cuz at the beginning like I didn't think it was that challenging of a class. Then stuff started to ramp up, and I was kind of falling behind for a little bit, and like I wasn't that great at proving stuff, and like I said, at the first exam, I did. I felt super unconfident about it.

This was coded for enactive experiences (not devoting enough time to the class and "falling behind") serving as a negative source of self-efficacy information. More examples of sources of self-efficacy for the participating students are presented in the next section.

3.3 Evidence of Associations between Five Principles and Sources of Self-efficacy in Student Self-efficacy

We did not have a large enough sample size (N=19) of students who took both pre- and post-semester surveys to discuss statistical correlations between use of the principles and increased student self-efficacy for proving; however, we studied the relationship between instructor use of the five principles and student-identified changes in self-efficacy by studying the intersections of the coding described in 3.1.3 and 3.2.2. These comparisons are summarized in Table 7; each cell lists the students for which coding for instructor use a principle intersected with coding for source of self-efficacy (called an association), along with the frequency of that association in parenthesis. No coding for negative sources of self-efficacy intersected with instructor use of the five principles.

Table 7: Summary of associations coded between for five principle and sources of self-efficacy

Associations Coded (frequency in parenthesis)		**Four sources of Self-efficacy**			
		Enactive Attainments	**Vicarious Role-modeling**	**Verbal Persuasion**	**Physiological Reactions**
Five Principles	**Gestalt**	Fred(1), Fannie(2), Francisca(2)			
	Uncertainty	Fred(1), Fannie(1), Frank(1), Francisca(1)			
	Aesthetic				
	Scholarly		Fred(3), Fannie(2), Frank(1) , Francisca (1)		
	Free Market		Fannie(2)		

The following subsections (3.3.1-4) include quotes from the four students for each of the above associations. Section 3.3.5 illustrates cases where students described themselves serving as potential role models for others. Throughout these sections, interview quotes coded for **the five principles** are in **bold**, and *sources of self-efficacy* are *italic.*

3.3.1. Gestalt Principle and Enactive Attainments Three of the four students from the Fall semester described ways in which use of the Gestalt principle provided a positive source of self-efficacy through enactive attainments. For example, consider Fred's experience proving in the course:

> *Fred*: It's a roller coaster of a class. You reach points where you're so frustrated that you can't solve stuff, and then the satisfaction when you actually . . . You figure out how to do **a proof that you've been working on for a while**. There's really no more empowering feeling in the world. *You feel like you can do anything!* But yeah, it's the trials and tribulations. You'll struggle and then it's *figuring out how to use that struggle to achieve something*, in the future, using what you know doesn't work and like, "all right, this doesn't work. Let's try to think of something new that might work better."

This description shows evidence of "aha" experiences – the satisfaction of "figuring out how to do a proof that you've been working on for a while" – which we coded for implicit use of the Gestalt principle since the instructor was assigning problems that allowed or required this approach to proving. This segment was also coded for enactive experiences because Fred appeared to be empowered by his success in proving and considered using this experience in the face of future difficulties. Thus, struggling on difficult problems and eventually proving them contributed to Fred feeling that he could "do anything," serving as a strong indication of his future ability to prove.

3.3.2 The Uncertainty Principle and Enactive Attainments All four students from the fall semester described teacher actions for the uncertainty principle that were coded in association with increased self-efficacy via enactive attainment.

Interviewer: How did the environment influence your learning to prove and you're gaining confidence in class?

Frank: Confidence? [The fall semester course] helped me **see where a lot of pitfalls were**, *and be okay with that, but also learn to anticipate those.* The ability to anticipate those was something that was pretty valuable, I think.

Being helped to see to see where his mistakes in proving were, and be fine with them, was coded for implicit use of the uncertainty principle since Frank connected the classroom environment with ambiguity and uncertainty in the proving process. Additionally, his learning and being able to anticipate his mistakes were coded as enactive experiences.

Francisca described being challenged "every single step of the way, [this class] challenged your thinking and how you approached math." Then the interview shifted to how being challenged influenced self-efficacy.

Interviewer: How do you think that contributed to your confidence in learning to prove?

Francisca: At first, it was nerve-racking cuz I wasn't getting things right, and I wasn't understanding things. But over the semester and over time, I actually talked to a couple of people about this: it was like, "**you don't have to be right in this class, cuz no one's gonna be right**." **There's like no concept of being correct**, and once you take away the idea of being correct or being right, *it makes your confidence level go up a lot more, cuz you're like, "I know that I did this, and this is what I accomplished, and so I should be proud of the work that I've accomplished."*

Not having to be "right" and not considering what is the one "correct" way of doing things demonstrates implicit use of the uncertainty principle, helping students become comfortable with ambiguous, open ended, or ill-posed problems. Interestingly though, at least part of her realizing this came from her peers, evidence of vicarious role modeling. This quote shows how this shift in perspective helped Francisca reframe her own perspective of her accomplishments, allowing her experience served as a potential source of self-efficacy, not necessarily in the information directly, but the weight she gives to these or future accomplishments in making self-efficacy judgements.

3.3.3. The Scholarly Principle and Vicarious Role-Modeling In the fall semester, all four students described teacher actions for the scholarly associated with positive sources of self-efficacy via vicarious role-modeling. The example given at the end of the Methods section is one such case. The following example, later in Fred's interview, was also coded for the scholarly principle and *vicarious role-modeling*:

Fred: When we would do the **peer discussions in class**, I would see how somebody else did it, and then I would be like, "okay that makes a lot of sense, how do you played around with it, and how you got to where you went" … I would, a lot of times, go back and … redo the two homework problems. *And thinking of how the other person solved it, and then that really helped me foster ways of being more creative, as I've said, using other people [and] how other people's work is creative, as a stepping stone for how I could be more creative.*

The response in italics was coded as a source of self-efficacy because Fred have previously been asked, “What do you think contributed to your gaining confidence in proving?” and responded, “by becoming more creative.”

3.3.4 Free Market Principle and Vicarious Role-Modeling All four of the student interviews from the fall semester were coded for use of the free market principle. For Fannie, the free market principle was coded in association with gaining self-efficacy via vicarious role modeling.

Interviewer: What in class contributed to your building confidence?

Fannie: The general **environment of everyone not being afraid to fail,** and I think the productive failure thing kind of contributed to that. Just generally **understanding that my peers weren't going to judge me for doing something wrong** was really refreshing. That was nice. And definitely having that time to work with other people was really important, because everyone kind of had their own perspective or their own different take on the problem… *Someone next to you might have had like a different idea about it that's just as correct as yours*…

This was coded for implicit use of the free market principle because Fannie knew “her peers weren’t going to judge her.” It was also coded for the scholarly principle due to Dr. F allowing students to engage and understand one another’s approaches to problems. Finally, because Fannie attributed her gaining confidence to being able to work with others without fear of judgment, this was coded for vicarious role-modeling.

Fannie also described the importance of “hearing other’s thought processes” and of using one another’s “individual strengths to come together to understand this problem and like make this proof” in relation to gaining self-efficacy for proving. Both these statements were both coded for the scholarly principle and gaining self-efficacy vicariously (3.3.3). However, immediately following this, she described implicit use of the free market principle through the way she experienced the environment of the class.

Fannie: I also liked that **there wasn't any like super overpowering voices** in the class, because I think that might have just been a characteristic of the people in the class, or it might have been the environment… I'm not really sure. But I know that I get super intimidated when there's just one person that's constantly dominating the conversation and I think that would have made me much more hesitant to *speak up or present my proofs*. So that was kind of nice: really understanding from Day 1 that no one was going to the judge you for failure, that was a really important part of the class.

From the classroom observations, we saw Fannie’s speaking up and presenting her own proofs in class as evidence her gaining self-efficacy from other students.

3.3.5 The Principles and Vicarious Influences Toward Others Although we initially set out to code the interviews for sources of self-efficacy for students in the course, we noted two specific cases in which some principles may have encouraged students to provide sources of positive self-efficacy to other students outside the course.

Interviewer: Is there anything else you gained from class?

Fannie: I don’t know. The ability to annoy my friends with math concepts. I was studying for my physics test the other day, and I went up to my friend, and was like “this is so cool” and it was one of the problems from my last test. I was like, “you’ve got to hear this. There’s these things called trapezoid numbers, and they're so cool.” And *I wrote it out on the chalk board*, and they’re like, “okay.” I’m like, “It’s cool. Numbers are cool!” But I don't know. *I did gain a lot of confidence*. Ultimately that was the biggest thing. Because you know, at the beginning I was like "eh, I don't know." But, towards the end, *I was like, "I can prove things. I can do it!"*

Even Fannie showed gaining an appreciation of something new (trapezoid numbers), this was not coded for the use of aesthetic principle since there was no mention of her instructor. However, because she was compelled to explain it to her friends with confidence, becoming a potential source of self-efficacy to her friends, we coded this for vicarious influences toward others. Francisca had a similar experience.

Interviewer: How long did you spend on homework?

Francisca: So much time. I was like "oh it's a [sophomore level] course. It won't be…" Oh my God, so much homework, so much time. I would spend like hours. My roommates would come home, and I would be doing a problem, they'd go back to class and come back, and I'd still be doing the same problem. And they're like "why? We've been gone for two, three hours and you're doing the exact same thing." …They also thought everything that I talked about for the whole semester was just absolutely crazy. I would bring up all the terms that we would use in class like "productive failure" and all the other things, and they're just like "you're nuts." *I was like, "no no no."*

This showed that Francisca's experience of the Gestalt and free market principles may have contributed toward her speaking out to her roommates, even in the face of rejection of her ideas, demonstrating how the principles may have encouraged Francisca to become a potential source of vicarious role-modeling toward others.

4. Discussion

The first goal of this project was to develop methodologies for studying the relationship between teaching actions for fostering mathematical creativity and changes in student self-efficacy for proving. As we've demonstrated in Section 1, there is significant literature on fostering mathematical creativity, yet little research studying the effects of creativity-fostering on students (Tan, Li, & Rotgans, 2011). We suspect this is, in part, due to a lack of development of effective or efficient research methods for studying the impact of creativity-fostering instruction. The methodologies developed in this study are one step towards developing this kind of investigation. We also hope this work motivates and provides direction toward developing explicit characterizations of creativity-fostering instruction and their potential effects.

Our second goal was to develop a better understanding the possible effects of creativity-fostering instruction on student self-efficacy. We cannot claim that the whole class gained self-efficacy for proving, nor that the instructor's actions for fostering mathematical creativity in the classroom effected the whole class. However, we show, from the student interviews, the specific ways in which the students described gaining or losing self-efficacy in association with instructor use of the five principles.

4.1 Discussion of Methodology

4.1.1 Construction and Implementation of Online Surveys To investigate instructor actions for fostering mathematical creativity, we attempted to find an appropriate framework with teaching actions that can be observed and characterized. Sriraman's (2005) five principles, while not always providing specific examples of creativity-fostering teaching actions, gave us a theoretically-sound framework that was domain-specific and aligned with our definition of creativity. This framework proved applicable for our provisional qualitative coding, as well as for gathering evidence of student experience of creativity-fostering instruction via the online 5PS survey.

The 5PS provided evidence that the majority of students experienced each of the five principles at least 6-10 times per semester (see section 3.1.2). However, in comparing student experience recorded in the 5PS with classroom observations (figure 1 and 2), their self-reported experience of the principles was less frequent than what we recorded from class observations. This highlights the discrepancy between student experience and teacher-use of the principles. It is logical that students are not consciously aware of each time their instructor employs a principle in class. Neither need they be for it to impact them (Rodiger, 1990). Thus, for future studies, we recommend rewriting the five principles survey in terms of instructor actions instead of student experience and administering this survey to both students and instructors. If properly aligned with classroom observation protocols, such a survey may be a more efficient and reliable way to measure use of instructor actions in the classroom (Hayward, Weston, and Laursen, 2018).

The self-efficacy for proving survey (SEPS) appears to be a robust instrument for measuring self-efficacy for proving. The Cronbach's alpha reliability estimates of the SEPS were 0.92, 0.90, and 0.92 for Surveys 1, 2, and 3, respectively ($\alpha \geq 0.9$ is excellent), indicating that the SEPS is measuring one construct. For surveys 1 and 2, students rated lower mean self-efficacy for more difficult proving statements, providing evidence of discriminative validity. We also observed in the task-based interviews that higher student self-efficacy ratings corresponded with greater success in proving, an indication of predictive validity. Yet, we believe some improvements can be made to the SEPS; for example, the higher mean and standard deviation of ratings for proving statement 2 on survey 1 may be related to the task's word length. Thus, for future use of the SEPS, we recommend using proving statements that are brief, such as those used in survey 2 and 3. This will allow students with broad mathematical backgrounds to more easily and accurately understand and evaluate their abilities related to the task.

Separate from the statistical significance of the SEPS, we believe the construction of the SEPS warrants more consideration in the field of mathematical education research. The SEPS was created following Bandura's (2006) recommendations: "the construction of sound efficacy scales relies on a good conceptual analysis of the relevant domain of functioning. Knowledge of the activity domain specifies which aspects of personal efficacy should be measured" (p. 310). We attempted to construct our scale utilizing domain-specific literature about the proving process (e.g., Hsieh, Horng, & Shy, 2012). Furthermore, the gradation of difficulty via Selden and Selden's (2013) continuum further allows the SEPS to capture a wide range of proving performances. Incorporating both research constructs provided us with a better, more precise understanding of self-efficacy than existing general self-efficacy scales in mathematics. For example, Iannone and Inglis (2010) utilized a self-efficacy for proving scale with statements such as "I am good at writing mathematical proofs." Earlier studies have shown that there are differences between algebra and analysis proofs (e.g., Dawkins & Karunakaran, 2016; Savic, 2017). Therefore, general statements about self-efficacy may not orient students to a reliable mathematical context for which they are gauging their own ability.

For future larger-scale quantitative studies, the pre- and post-SEPS and modified (as described above) end-of-semester student and instructor 5PS could be used to study the correlation between specific teaching actions and changes in student self-efficacy for proving. In particular, hierarchical linear modeling (Raudenbush & Bryk, 2002) could be used study relative or combined influence of specific teaching actions on changes in student self-efficacy for proving.

4.1.2 Qualitative Methodology via Observations and Interviews The classroom observations provided a way to document instructor use of the five principles (see section 3.1.2). We can see that, throughout the course, Dr. F utilized both explicit and implicit teaching actions that align with the five principles (see Table 2). The actions shown in 3.1.2 provide examples that further strengthen Sriraman's (2005) conjectured creativity-fostering in the classroom.

Some actions were also apparent due to the nature of the pedagogy. Dr. F utilized inquiry-based teaching, which may have played a significant role in how the scholarly and free-market principles were used. However, there are ways in which an instructor can use lecture-based pedagogy with a saturated emphasis on mathematical creativity. Omar et al. (2018) described one primarily lecture-based professor who used reoccurring assignments with open-ended questions and written reflection on their problem-solving process, assigning more grade-weight to the reflections. In this class, students transitioned into feeling "more like mathematicians" which may correspond to an increased sense of self-efficacy. We recommend for future studies to expand the methods used in this study for different pedagogical approaches.

One limitation of the methods used in this study is that we only considered creativity-fostering instruction *within* the classroom. According to Sriraman (2005), these principles "can be applied in the everyday classroom setting" (p. 26). However, we observed several cases in which factors outside of the classroom appeared to determine how students were influenced by the principles. For example, Dr. F's office hours appeared to have had an impact on Francisca's self-efficacy. When asked "Is there anything else Dr. F did that influenced your confidence for proving?" she replied:

> I think it was just how open he was. It was just easy to go in and approach, and like he was just like "if my doors open, just come in, and we'll talk about it for hours." It was just easy to make an appointment and like you could talk for an hour, you start about math and talk about something completely different. And that was really nice.

While this action was not coded for any of the five principles since it did not occur within the class, it illustrates how the instructor's office hours created a secondary environment where the principles may have been enacted one-on-one. Additionally, it demonstrates a context where verbal persuasion may have had a greater influence on building student self-efficacy due to the rapport Dr. F built with Francisca.

Francisca's interview also illustrated how working together with her peers outside of class may have influenced her self-efficacy. Francisca said being "challenged every single step of the way, challenged your thinking, and how you approached math" contributed to building her confidence. When asked "how?", she replied:

> At first, it was nerve-racking cuz I wasn't getting things right, and I wasn't understanding things. But over the semester and over time -- I actually talked to a couple of people about this: it was like, "you don't have to be right in this class, cuz no one's gonna be right."

> There's like no concept of being correct, and once you take away the idea of being correct or being right, it makes your confidence level go up a lot more, cuz you're like "I know that I did this and this is what I accomplished, and so I should be proud of the work that I've accomplished [enactive attainment] rather than whether or not the work that I accomplished is 100% correct, and I'm 100% doing this by the book.

Again, this was not coded either the uncertainty or free market principles because it likely occurred outside of the class; however, this illustrates how norms related to use of these principles were likely enacted or reinforced through conversations students had *outside* the class. Therefore, the way students are assigned to work outside of class appears to play a role in how the principles influence students' self-efficacy. Other avenues to better understanding the role of the five principles play outside the class may include studying the type of problems assigned (uncertainty principle) and how problems are assigned, graded, or revised (Gestalt principle).

In the student interviews, there was only one instance (Francisca) of students describing gaining self-efficacy via verbal persuasion, and no instances of physiological reactions serving as a source of self-efficacy. This may be a result of the way interviews were conducted. The interview focused on students experience in class overall (see Appendix C) and did not explicitly focus on one-on one interaction between student and instructor, nor the way students individually felt in class. We also did not have a way to more directly measure students emotional and physical reactions in class, which can significantly impact student self-efficacy (Bandura, 1997). It is possible that the scholarly principle can be related to students developing a sense of community and belonging in class, and in turn, help foster a sense of personal security or comfort in class. At the same time, engaging in class, or presenting one's ideas (such as through the "productive failure assignment") may also serve as a source of anxiety or fear, and thus, a negative source of self-efficacy. This would further highlight the importance of instructors attending to the scholarly and free market principles together when employing the scholarly principle.

Additionally, we recommend developing ways to investigate the quality of implementation the five principles. We conjecture that use of a principle does not behave like a binary variable in relation to student development, but rather, that specific factors enable or strengthen the impact each principle has on students. For example, consistent personal value placed on productive failure assignment throughout the class may have gone farther in showing students it is safe to take risks than simply saying "I want you to take risks." Fannie (in section 3.3.4) stated as much in her response to gaining confidence, "The general environment of everyone not being afraid to fail**,** and I think the productive failure thing kind of contributed to that [gaining confidence]." We conjecture that qualitative analysis of instructor beliefs, values, or goals pertaining to the use each principle, as well as the alignment of those beliefs, values, or goals with teaching actions, may be one way to infer the quality or stability of their use.

4.2 Relationship between five principles and ways students gain self-efficacy

The methods used in this study – classroom observations, surveys, and student interviews – all provided evidence of instructor use of the principles within the classroom. The SEPS also measured a statistically significant change in students' self-efficacy. These results provide a context for which we can begin to answer the second research question: how does instructor use of creativity fostering teaching actions impact how students gain self-efficacy for proving in an introduction-to-proofs course?

Coding the interviews provided evidence that the four students experienced changes in self-efficacy for proving occurred in relation to instructor use of the five principles in the following two ways:

1. Teaching actions coded for Gestalt and uncertainty principles were associated with increased self-efficacy via enactive experiences, and
2. Teaching actions coded for free market and scholarly principles were associated with increased self-efficacy via vicarious role-modeling.

Here, we reiterate that these results were found in only the students that were interviewed and not the class as a whole. In the following subsections, we discuss the implications of these associations (4.2.1 and 4.2.2). Then, we offer some explanation for why the influence the aesthetic principle, verbal persuasion, physiological reactions may not have reported (4.2.3).

4.2.1 Gestalt and Uncertainty associated with Enactive Experiences In our qualitative analysis, we observed that all four participants cited instructor actions associated with gaining of self-efficacy for proving. In particular, the Gestalt and uncertainty principles were associated with gaining self-efficacy via enactive experiences. Fred's experience (see section 3.3.1) illustrated this well: "You reach points where you're so frustrated that you can't solve stuff, and then the satisfaction when you actually . . . you figure out how to do a proof that you've been working on for a while. There's really no more empowering feeling in the world. You feel like you can do anything!" Through Fred's experience, we saw Gestalt and uncertainty principles were intertwined: Fred was both given time from the professor to work on challenging proofs and was subsequently uncertain because Dr. F did not provide any proof to Fred. While giving Fred the answer or proof to a statement may have helped Fred's understanding in the short term, it would not have given him the opportunity to build self-efficacy via his own Aha! experiences (Savic, 2016).

Fostering creativity by employing the Gestalt and uncertainty principles to build student self-efficacy may help mitigate students' difficulties with proving, such as failure to explore for new ideas in proving, failure to rework an argument in the case of a suspected error, and failure to validate a completed proof (Selden & Selden, 2010). Researchers have found students' problems in learning to prove can include an inability or "unwilling[ness] to generate and use their own examples," and "not know[ing] how to begin proofs" (Moore, 1994, pp. 251-252). In Furinghetti and Morselli (2009), the student Flower attempted the proof once and stated, "'Help! I cannot do it, I still do not see anything. The deepest darkness'" (p. 81). This is in contrast to Fred: after gaining success in proving difficult statements, he was empowered with a sense that he could do anything, and thus is more likely to exhibit persistence needed in proving. Thus, we challenge instructors to consider the Gestalt and uncertainty principles when planning activities, posing problems, and engaging student participation in class.

4.2.2 Free Market and Scholarly associated with Vicarious Role-Modeling The second main result of our investigation was that the free market and scholarly principles were associated with vicarious role-modeling as a source of self-efficacy in proving. Fannie (section 3.3.2) described gaining confidence in class from learning and building from her peers' ideas, evidence of use of the scholarly principle. Immediately following this, Fannie described also gaining confidence as a result of "the general environment of not being afraid to fail," evidence of implicit use of the free market principle. Failure was then modeled in a way that promoted learning and intellectual growth, which can have a continuing impact on how students gain self-efficacy in future proof-based courses (Savic, Gunter, Curtis, & Paz Pirela, 2018).

Furthermore, it wasn't just the "productive failure" component of class that appears to have fostered student risk-taking; from the class observation (section 3.1.2), Dr. F took an answer from a student that could have been deemed wrong and reframed it as a subset of the set of correct answers. This microcosmic action can have an effect on student risk-taking; the next person may be less afraid of contributing and building off others' ideas if they know that their contribution will be valued by the instructor. In turn, there is greater potential for students to gain self-efficacy from peers of perceived similar ability. Also, from the classroom observation (section 3.1.2), we observed Dr. F offering Fannie the opportunity to lead class discussion. This action demonstrates how the free market and scholarly principles can work in coordination. Fannie being comfortable enough in class to take the risk of leading discussion led to her being able to engage students in discussion and debate over the classroom material.

This result is particularly significant in light of some of our findings from the summer preliminary data collection. One student, Sam, described vicarious role-modeling from his peers as a *negative* source of self-efficacy:

Sam: A lot of times [Dr. S] would introduce a new problem and tell us to work on it… There [were] times when he would engage the class like earlier on in the semester and I felt comfortable about like speaking up and answering occasionally, but a lot of the time I didn't feel comfortable around my peers to answer questions.

Interviewer: Do you think your confidence of varied depending on the subject, or how did you become more confident by the end? Because you said [earlier that] you were confident.

Sam: …In this class setting I felt like there were people in this class that already knew, like there's like two people in particular, that would always answer all the questions and … I just deferred the questions to them, so if the teacher posed a question to the class and they didn't answer it, then *I felt it like "well, I definitely can't answer it if they can't*."

We coded this for explicit use of the scholarly principle because the instructor was posing problems and giving students opportunities to contribute to and extend the classroom community's body of knowledge. However, Sam feeling like "*I definitely can't answer it if they can't,"* was coded as a negative source of self-efficacy. We also noted that throughout Sam's interview the free market principle was not coded; he did not cite any way in which the instructor encouraged risk taking or provided an environment where the student felt safe to take risks. This *negative* source of self-efficacy cited by Sam corresponds with Bandura's (1997) observation that those "observing others perceived to be similarly [or more] competent fail lowers observers' judgment of their own capabilities and undermines their effort" (p. 87). Sam's response appears to be in direct contrast to Fannie who felt that her "peers weren't going to judge me for doing something wrong." These differences demonstrate that evidence of the five principles do not always provide *positive* sources of self-efficacy, and that instructors must be aware of this if they consider implementing the scholarly principle in their classroom.

4.2.3 Potential role of the aesthetic principle, verbal persuasion, physiological reactions

Although both the classroom observations and online surveys demonstrated both classroom presence and student experience of the aesthetic principle, only Frank mentioned one teacher action for the aesthetic principle. Fannie was the only student interviewed who described appreciation of the beauty or elegance of mathematical ideas in relation to other students, not the instructor. In section 3.3.5, Fannie stated one of the problems from her last test was “so cool” and wrote it out for her friends to look at instead of studying for her physics test. At the same time, observation of instructor use of the aesthetic principle was no less scarce then the Gestalt principle. It may be that instructor use of the aesthetic principle influences student self-efficacy indirectly by promoting interest, motivation, and engagement, which may mediate the influence of the other four principles on self-efficacy. For example, increased interest may contribute to students engaging difficult proofs, persisting in the face of uncertainty, building on one another’s ideas, and taking risks (Ryan & Deci, 2000).

Additionally, only instance of students gaining self-efficacy via verbal persuasion (Francisca) and no instances of students gaining self-efficacy via physiological reactions were coded in the student interviews (see tables 2 and 3). This may be a result of the way interviews were conducted. The interview focused on students experience in class overall (see Appendix C) and did not explicitly focus on one-on one interaction between student and instructor, nor the way students individually felt in class.

We also did not have a way to more directly measure students emotional and physical reactions in class, which can serve to impact student self-efficacy (Bandura, 1997). It is possible that the scholarly principle can be related to students developing a sense of community and belonging in class, and in turn a sense of security when in class. At the same time, engaging in class, or presenting one’s ideas (such as through the “productive failure assignment”) may also serve as a source of anxiety or fear, and thus, a negative source of self-efficacy. This would further highlight the importance of instructors attending to the scholarly and free market principles together when employing the scholarly principle.

This said, it is also likely that verbal persuasion and physiological reaction had a relatively small impact on students’ self-efficacy for proving. According to Bandura (1997), “verbal persuasion alone may be limited in its power to create enduring increase in perceived efficacy” (p. 101). Additionally, physiological reactions tend to carry more generalized effects on self-efficacy and are more relevant to domains that involve greater demands on physical functioning.

4.3 Future Research/Further Theoretical Considerations

Although we initially set out to find the ways in which the five principles impact student self-efficacy, we recognized in the course of this research, several other factors that appear to influence the impact certain principles have on student self-efficacy for proving. Firstly, evidence of the association between the five principles and students serving as role-models toward others outside the classroom highlight the importance of these principles in changing students’ attitudes toward mathematics. Fannie and Francisca actively engaged their peers in a way that demonstrated their care for the subject (section 3.3.5), reflecting of a change in their own self-perceptions, or identities, concerning mathematics. Several studies have found connections between creative identity and strong self-efficacy for creative attainments (Jaussi, Randel, & Dionne, 2007; Karwowski et al., 2013; Tierney & Farmer, 2011). We suggest investigating how the use of the five principles in creativity-fostering impacts students' mathematical identities.

We also observed two other psychosocial mechanisms that may have influenced how the five principles may have impacted student self-efficacy for proving: the negotiation of classroom norms (Rasmussen et. al, 2015) and integration of new mathematical standards (Bandura, 1986). We believe that classroom norms of challenging and building on one another's ideas (section 3.3.3-4, 4.2) afforded by the scholarly and free market principles may have allowed students to better internalize mathematical standards necessary to proving. If the primary external rewards (affirmation of a proposed proof, engagement of ideas by the class) are intimately related the mathematical discourse, they may provide standards for which students strive and boost competence when they are attained (Bandura, 1986). Thus, presence of classroom norms related to the scholarly and free market principles may moderate the effect of the principles on self-efficacy.

In turn, we conjecture that engagement in the Gestalt and uncertainty principles provide students with opportunities to testing and developing their own system of standards, internalizing their own sense of efficacy and motivation in proving. Both one's own experiences proving, together with comparison of those attainments within the classroom dialogue, can serve to build standards for *measuring* one's own proving progress and *evaluating* future self-efficacy. Because of this, we believe studying the interaction between classroom norms, personal standards, and student motivation may play a central role in understanding the impact fostering creativity can have on student self-efficacy for proving.

This work may also contribute to previous inconsistent findings on the influence of classroom climate on student self-efficacy. Studies of the primary (Salinas & Gar, 2010) and upper secondary school (Fast et al., 2010) have demonstrated a positive impact of learner-centered classroom climates on students' self-efficacy and achievement, while one study of tertiary students (Peters, 2013) found that learner-centered classroom climates had lower mathematics self-efficacy levels. However, in this study, we observed an inquiry-based, student-centered classroom fostering significant gains in mathematics self-efficacy. This highlights the need for understanding of the potential intersections of creativity-fostering and inquiry-based classrooms. For instance, to what degree was use of the free market and scholarly principles a natural result of the classrooms being inquiry-based? How do other inquiry-based classrooms handle the Gestalt and uncertainty principles? The tools offered in this study may advance further studies of the impact of instructor actions, and thus can lead to a better understanding of the kind of classroom environments that can best foster student development in mathematics.

5. Conclusion

In this paper, we explored how fostering mathematical creativity in a tertiary introduction-to-proofs course may impact self-efficacy for proving. First, we developed methods grounded in previous theory of fostering mathematical creativity (Sriraman, 2005), proving (Hsieh, Horng, & Shy, 2012; Selden and Selden, 2013), and self-efficacy (Bandura, 1997; 2006) to study both creativity-fostering teaching actions and student self-efficacy for proving. Through an online student survey (5PS), classroom observations, and student interviews, we documented instructor use of teaching actions that aligned with all five principles for fostering mathematical creativity. We also developed and tested an instrument for measuring student self-efficacy for proving (the SEPS) which recorded a statistically-significant increase in student self-efficacy for proving from the beginning and end of semester. We provided several recommendations for future improvement and use of the methods developed in this study.

Next, we analyzed how the use of the five principles in the classroom may be related to the ways in which students gained or lost self-efficacy. Analysis of the four students interviewed provided evidence that these students gained or lost self-efficacy via enactive experiences and vicarious role-modeling in relation to instructor use of four of the five principles. In particular, the instructor fostering freedom of time and space to work on challenging problems (Gestalt principle) and exposing students to uncertainty in mathematics (uncertainty principle) appeared to support students' gaining self-efficacy via their own attainments. Additionally, the instructor allowing students to build off one another's ideas (scholarly principle) appeared to promote students gaining self-efficacy from their peers. However, without the instructor creating an environment where students could take risks (free market principle), we conjecture students would have been less likely to view their peers' participation as a positive source of self-efficacy. The four cases studied in this paper illustrate why instructors should consider these pairs of principles in coordination (Gestalt and uncertainty; scholarly and free market). Together with research already showing the strong influence self-efficacy plays in predicting student performance (Siegel, Galassi, & Ware, 1985; Pajares & Kranzler, 1994), the results of this research can provide direction for instructors in supporting the development of students' abilities for mathematical proving.

Finally, this study may provide a new perspective for researching the effects of mathematical creativity in the classroom. It highlights the impact fostering creativity might have on constructs not frequently associated with creativity. Perhaps one goal of mathematics education, in addition to developing pedagogies that enable students to be creative, is to better understand the residual effects of students' *belief* in their own creative potential. We conjecture that when students *feel* creative, i.e. believe in their own creative potential, constructs such as self-efficacy are impacted, which can have lasting effects on students' long-term mathematical trajectory.

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Appendix A: Five Principles Survey (5PS)

Instructions: Answer the following question based on this **current semester's class**.

0	1	2	3	4	5
Never	1 to 2 times per semester	3 to 5 times per semester	6 to 10 times per semester	Weekly	Daily

_____ 1. How often did you have the freedom (of time and space) to work on a challenging problem or proof over a period two or more days?

_____ 2. How often did you experience the joy of arriving at a solution after working on a problem or proof for several days?

_____ 3. How often did you find yourself appreciating the beauty or novelty of creating new mathematical ideas or solutions?

_____ 4. How often did you find yourself appreciating the beauty or novelty of creating new mathematical ideas or solutions?

_____ 5. How often did you feel comfortable going in a direction on a problem/proof that may or may not prove successful?

_____ 6. How often did you enjoy engaging new or atypical thinking in your approach to solving a problem?

_____ 7. How often did you engage in debate with your peers or instructor concerning how to approach a problem?

_____ 8. How often did you challenge the validity of your peers' or your instructor's solutions/proof?

_____ 9. How often did you feel comfortable considering ambiguous or ill-posed problems?

_____ 10. How often did you feel comfortable working on open-ended or potentially unsolvable problems?

Notes:

- Questions were randomized.
- Questions 1-2 correspond with the Gestalt principle
- Questions 3-4 correspond with the aesthetic principle, and were repeated for validation purposes
- Questions 5-6 correspond with the free market principle
- Questions 7-8 correspond with the scholarly principle
- Questions 9-10 correspond with the uncertainty principle

Appendix B: Self-efficacy for Proving Survey (SEPS), Survey 3

Instructions: For the next three questions, please rate your confidence in your own ability to do the following sub-tasks related to creating a proof, or logical argument, of three mathematical statements. However much you may want to prove these statements, please do not! Just evaluate your own confidence in your ability for each of the sub-tasks.

What is your confidence in your ability to do each sub-task?

0	10	20	30	40	50	60	70	80	90	100
Cannot do at all					Moderately certain can do					Highly certain can do

Statement 1: If $x, y \in \mathbb{R}$, then $|x + y| \leq |x| + |y|$.

_____ Understand and informally explain why a statement is true or false
_____ Explore new ideas to come up with ways to start your proof
_____ Use various representations (numbers, pictures, tables, words) to structure your thinking
_____ Formally write out and justify each step of your proof
_____ Examine your proof for accuracy and identify any missing steps

Statement 2: If n is an integer, then $1 + (-1)^n(2n - 1)$ is a multiple of 4.

_____ Understand and informally explain why a statement is true or false
_____ Explore new ideas to come up with ways to start your proof
_____ Use various representations (numbers, pictures, tables, words) to structure your thinking
_____ Formally write out and justify each step of your proof
_____ Examine your proof for accuracy and identify any missing steps

Statement 3: Every odd integer is the difference of two squares.

_____ Understand and informally explain why a statement is true or false
_____ Explore new ideas to come up with ways to start your proof
_____ Use various representations (numbers, pictures, tables, words) to structure your thinking
_____ Formally write out and justify each step of your proof
_____ Examine your proof for accuracy and identify any missing steps

Appendix C: Student Interview Questions

*Students were first given 30 minutes to prove the three proof statements from Survey 3.

Task:

1. Tell me about your proving process?
2. What tools did you use?
3. Have you seen any of these proofs before?

Class Experience:

4. Tell me a little about your experience in this class.
    - Describe your experience of learning to prove mathematical statements.
5. What is your definition of mathematical creativity?
    - Did you think you were creative in this class?
6. At the beginning of this semester, how confident were you in proving mathematical statements.
    - What about now?
7. What do you think contributed to building your confidence?
    - How did the homework contribute to your gaining confidence in proving?
        - What is the longest you worked on a problem?
        - How did you approach open-ended problems?
    - How did your instructor influence your learning to prove?
        - Class discussion
        - Working in groups
8. What would have helped you build your confidence in proving?
9. Where did you struggle the most in this class? What prevented you from gaining confidence?

Additional:

10. What would you have changed about this class?
11. Overall, what did you gain from this course?
12. What else would you say about your experience in this class?

Appendix D: Self-efficacy for Proving Survey (SEPS) Results

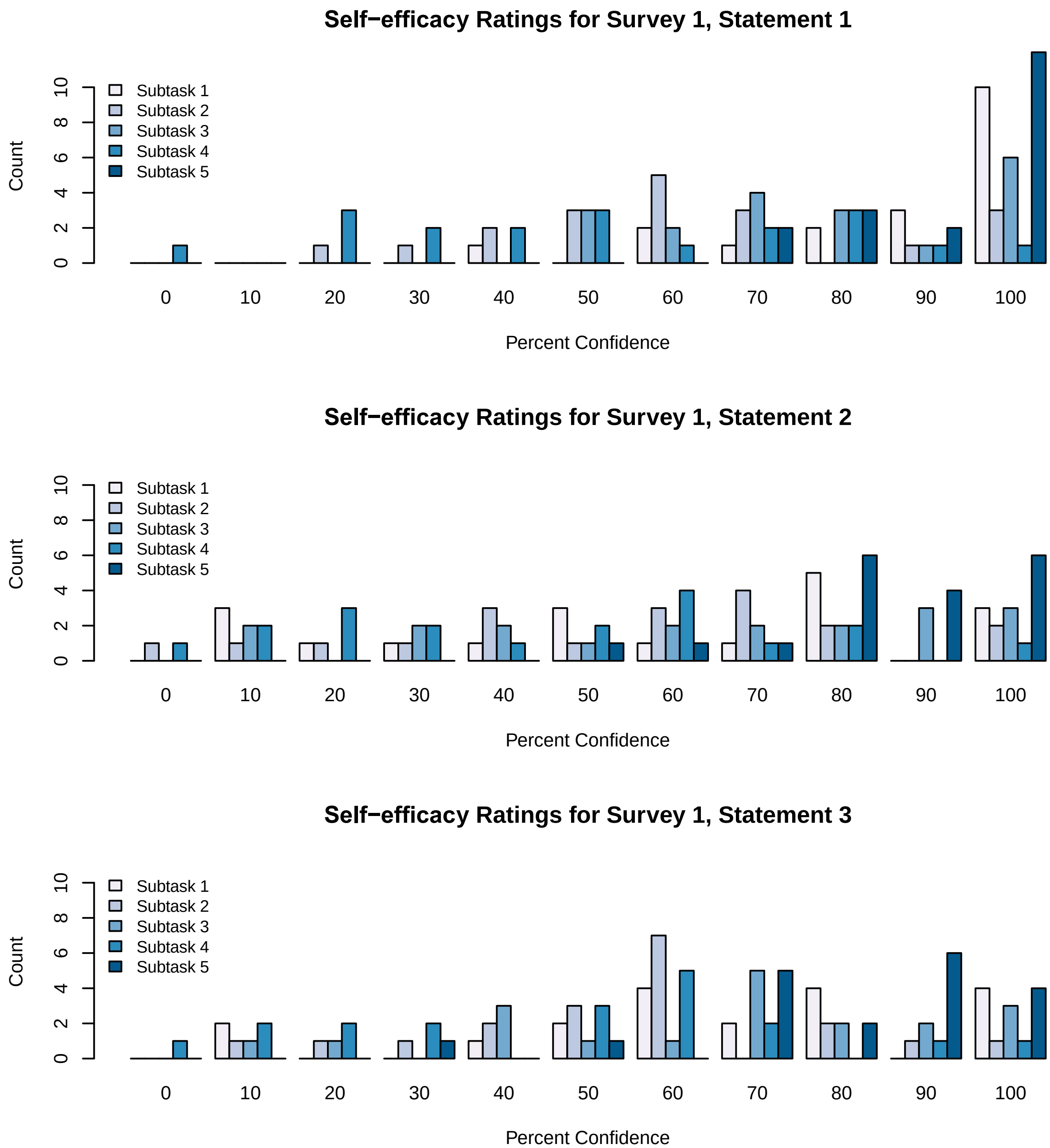

Figure D.1: Beginning of Semester SEPS ratings

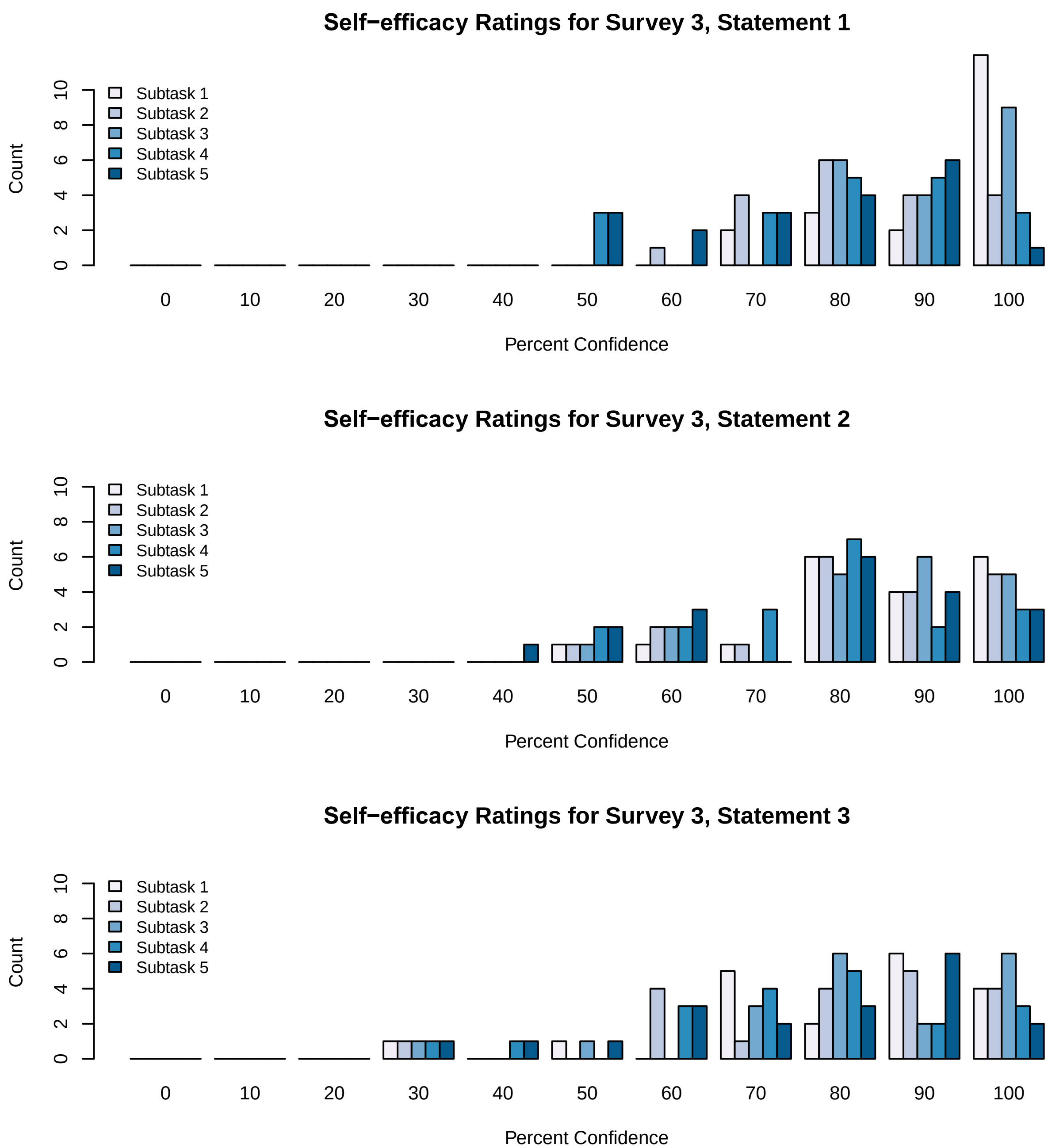

Figure D.2: End of Semester SEPS ratings