Introduction

Math education research interests many faculty, both with and without formal backgrounds in math education. Faculty with more traditional backgrounds may have studied math education in graduate school and then pursued post-doctoral work in the field before obtaining faculty positions. This prepares them for research through formal training in and practical exposure to ideas, theories, methods, and common practice within the field.

Many faculty however become interested in math ed after obtaining a faculty position (often, but not always a teaching focused position) (SFES citation). These faculty often do not have formal training in math ed, nor do they have much practical experience in the field. As a result they struggle to catch up with the field's understanding of theory, methods, and research practice more generally. These emerging math education faculty may have unique struggles with math ed as they attempt to set up a research program within the field. In this paper we focus on a particular area of concern, theory. In spite of attempts within the field to make theory more accessible (which we discuss more in the next section), theory remains a significant barrier to participation in math ed for emerging math ed faculty.

Many emerging math ed faculty express confusion or worry around the complexity and usage of what are often referred to as theory, theoretical frameworks, analytical frameworks, and theoretical perspectives. In this paper we discuss these terms and the literature surrounding them as part of our discussion of confusion among emerging education researchers. Here we present three ways in which emerging math ed faculty in the Professional-development for Emerging Education Researchers (PEER) program \cite{Franklin2018} struggle with theory, and present preliminary evidence that for some of these faculty learning about theory is transformative for their research and self-image.

Theory in Math Education

Many authors have made an effort to make theory more accessible (a fine introduction is here, for example \cite{shayan-doroudi}). Cresswell & Cresswell quote Kerlinger to provide a definition of theory within social sciences as "a set of interrelated constructs, definitions, and propositions that presents a systematic view of phenomena by specifying relations among variables, with the purpose of explaining natural phenomena" \cite{Cresswell, pg 95, Chapter 3}. Schoenfeld provides a rich discussion of what theory is in math ed \cite{Schoenfeld}. The ideas presented by Schoenfeld are similar to what Cresswell has to say, but provide a more nuanced view of how these ideas translate into mathematics education.

These definitions of theory are just the beginning, however. As argued by Stinson \cite{Stinson2020} our choices of theory and how we use it are strongly influenced by our philosophical perspectives and worldviews as researchers. Many other authors meanwhile have attempted to write concise introductions to the particular theories and theoretical frameworks which may exist, as well as attempting to distinguish between their various uses and how they

all fit into the broader literature \cite{Simon2009, Spangler & Williams, Lester2005, Silver & Herbst}. Even as these authors work to provide detailed introductions to the theory of mathematics education research they all recognize, as do we, that the roles theory plays in research can vary. This complex process of making decisions about what role theory plays in your research, as well as which theory or theories to use can be very confusing and intimidating to emerging math ed researchers.

Study Context: The PEER Program

This study was conducted as part of a grant for improving and expanding the Professional-development for Emerging Education Researchers (PEER) program (Franklin 2018). The PEER program brings together emerging education researchers at a workshop to do intensive writing and thinking about research questions and research design. The program also covers selected topics in research based on participants's needs (there may be workshop sessions focused on particular methods or theories for example, or on authorship). The bulk of the program centers on participants working together and sharing and refining research ideas together with the support of PEER coordinators. PEER coordinators intersperse groupwork activities with instruction and guided discussion around subject matter and research design.

Our data for this paper is drawn from participants in the PEER-Chicago 2021 field school. This field school occurred over Zoom through May and June 2021. It consisted of a kickoff session, three 2 hour workshop sessions spread over 6 weeks, and then a 3 day intensive at the end of June. Before the field school we conducted pre-interviews with 14 participants. During the field school participants were encouraged to record burning questions that they had in a padlet document during each session. We recorded these padlet documents. Finally we performed post-interviews with 8 participants following up on their experiences during the field school.

Research Perspectives of the Authors

There exist a wide variety of perspectives on what theory is, the various perspectives and pieces which contribute to it, and the ways in which it should be used, to say nothing of the wide diversity of theories themselves. The authors of this paper are conducting research on and are affiliated with the PEER program. The PEER program takes a parallel view of research, seeing the selection of theory, literature review, data analysis, writing, and formation and revision of claims as ongoing and often overlapping parallel processes. The different parts of research are not conducted linearly, but occur together and inform each other.

Theory is considered at all stages of a research project, and choices of theory and its place within a particular project are constantly deliberated, in line with Simon. We also agree with Stinson that personal worldviews and philosophical alignment play an important role in the selection and interpretation of theory and data. As we present our findings here, it is important to note that the PEER program teaches theory in this way: as something that is deeply and inextricably attached to peoples worldviews, personal and academic philosophies, and which

influences and is influenced by the other pieces of your research program. The authors subscribe to this view of theory.

Data Analysis and Discussion

In the pre-interviews discussion of theory by participants was rare. Since the interviews were semi-structured and focused on participants' experiences, the authors didn't force discussion of particular topics. Several participants did discuss theory during their pre-interviews, and we think it worth noting that all of these participants had prior experiences in projects conducted by math ed researchers. Ryan and Lily in particular piqued our interest with their comments on theory.

Ryan says: "[There are so many] *different theoretical perspectives that one can adopt when you're looking at your data, and that's another place where you sort of get this pigeonhole effect, right? Where you just kind of do what you know and if you don't know it, you can't do it.*" Which was an early confirmation that participants find theory overwhelming to confront. It also speaks to one of the ways that our participants struggle with theory. Ryan struggles with finding theory that is a good fit for his task, there are so many kinds of theory out there that if you don't know precisely what you're looking for or where to look for it, it's hard to find new theory.

Ryan goes on to say: "[W]*ith those theoretical perspectives is just to even to have, you know, a handful of references that I can go to in the literature and read some more about over the next, you know, three or four months* [would be very helpful]." This statement highlights the value of simply linking Ryan to the community's conversation on theory. Many EDBERs express a similar sentiment to Ryan, that simply having a few curated sources to look at would be highly valuable. Several examples of such sources can be found in the paper's citations, and were provided to participants at the workshop.

References may not be sufficient however. As our participant Lily said about theory: "... *[A]t the beginning, the whole idea of a theoretical framework was totally mind boggling to me."* and "*I was talking a lot with like my grad student friends at the time in sociology and things like this, and like that's their bread and butter...".* Lily highlights the importance of having someone to hold a conversation with and bounce ideas off of when trying to understand theory. Perhaps more interestingly for us she highlights a second way in which our participants struggle with theory: they aren't sure what theory and all of the terminology surrounding it means within math ed.

The scarcity of comments on theory in our pre-interviews was somewhat surprising to us as the interviewers often explicitly asked participants about theory in math ed research. Lily and Ryan perhaps provided us with our first glimpse into why interview participants so steadfastly avoided the topic: because it is overwhelming. The meaning of theory, and the web of terminology surrounding theory in math ed can be very confusing to emerging math ed researchers. Secondly, even for those who have developed an understanding of what theory is

in math ed, it can be very difficult to source theory and learn about new theories without guidance.

During the workshop theory became a focus topic due to the questions that participants had, as well as our observations from the pre-interviews. The data we present here is taken from participants' padlet questions. The padlet questions reflect both of the ideas we heard from Lily and Ryan: that emerging math ed researchers struggle to figure out where they should source theory, and that they struggle to understand what theory is/means in math ed. However we also see some additional ideas in the padlet data.

Within the padlet data we see some of the same confusion that Lily and Ryan had. Participants expressed plenty of confusion about where theory should be sourced. For instance one participant asks "How much of your theories should be based on other work (lit review) vs. your own new ideas?" It's common among participants to be uncertain about how their own ideas interact with theory in math ed research. Should theory come from other published research, or should it be developed by the researcher? Others express similar questions about where theory should derive from, and how to appropriately build one's own ideas into theory. One participant asks "How is it distinguishable from a lit review? Don't the theories come from published research?", and another asks "How does one *develop* a theory from data?".

It is clear to our participants that theory must be generated by someone somewhere, however they are uncertain about community norms around who is allowed to generate theory and how. The problem of where to source theory and how to do it appropriately is big stumbling block for participants.

Perhaps one of the most illuminating questions asked by a participant was "Does the "theory" for a paper necessarily need to be a complete ~theory~, or can it be a framework or even just a Frankenstein of ideas you were thinking about when looking at your data? Does every paper need a "theory"?" The first part of this comment echoes confusion that we had already heard about from Lily and Ryan. The asker is uncertain about what theory is, what theory means in math ed. They are also (as a consequence) uncertain about where it comes from, who makes it, and how. However, this participant is not merely confused about what theory means, or where it should be sourced. The second question here "Does every paper need a theory?" speaks to a deeper concern: what is the purpose of theory?

Another participant asks this explicitly: "Why do you even need a theory? This theory stuff is super intimidating, what's the best way to ease yourself into it?" Many of our new education research participants struggle to understand the role of theory in our research. What exactly is it that we use it for?

The questions asked by participants during the workshop reinforced comments by Ryan and Lily. Participants are confused about where theory comes from, and what theory means. However, the padlet questions also made clear a related confusion. Participants struggle to understand the role of theory in math ed research.

After the PEER-Chicago workshop we conducted follow up interviews with 8 participants. These interviews focused on their experiences during the PEER workshop, and the development of their understanding and perceptions of education research as well as their professional identity. Several participants spoke about the how learning about theory during PEER-Chicago had impacted their understanding of education research.

In discussing learning about theory Peter notes: "*The idea of using part of this theoretical framework and combining it with that one, I'd never considered it before*". Learning about theory at PEER transformed how Peter thought about implementing theories in his own research. It him resolve a long standing problem he had commented on earlier: "*I know enough about theoretical frameworks to know that there are constructivist theoretical frameworks, and you know there are varieties of those, and then there are other theoretical frameworks which are not constructivist. And I can read about both of those, and I can see what I'm doing in each, and that just doesn't make sense to me.*" Over the course of PEER his thinking on theories moved away from an image of theories as mutually exclusive, descriptions of the world to theories as potentially complementary descriptions of the world.

Another participant, Olivia discusses how learning about theory in math ed has transformed her view of math ed research: "*Well I think the most pointed, that first reading specifically addressed the value of a more theoretical approach to math education, how that can inform a more practical approach.*" Olivia goes on to say: "*I tended to come at things as I want to know if this works, I want to know how to tell if it's going to work. Which is implicitly dismissive of people who want to simply ask questions like, you know 'exactly what do students leave with from this particular approach to describing a logical construct?' and I am less dismissive off that [now].*" Olivia has become more open to questions which aren't directly about measuring classroom success, and curriculum implementation.

Finally, Penelope comments: "The theories, I still don't have a good handle on the theories, but I found that for me, [...] my focus is more like 'let's make this really good' versus some people who are like 'let's make theory really good' if that makes sense. And so I did get some understanding of where I am and why I'm there and being ok with why I'm there." Penelope's view of their own place in math ed research and the acceptability of what they do was transformed by taking the time to learn more about theory.

These comments from our post-workshop interviews suggest that for some participants learning about theory is transformative. It has a large impact on their views of math education, their identities within math ed, and their understanding of how to conduct research. It's very exciting for us to see that learning about theory can have a deeper impact on participants than simply providing a theory paragraph in their paper.

Concluding Remarks

Theory is deeply important to math education research. It informs research projects at all stages of development. A researcher's use of theory is heavily impacted by their own worldviews, personal philosophies, and experience. For many emerging math ed researchers theory is very intimidating, and difficult to approach or understand. In this paper we have explored several ways in which emerging math ed researchers participating in the PEER program struggle with theory. We found that these emerging researchers struggle with understanding what theory means in math ed, what role theory should play in their research, and where and how to source theory.

We also found that after participating in PEER a number of our participants discussed how learning about theory has impacted their engagement with math education. While it was unsurprising that some participants discussed the application and use of theory in their research, their discussion was not limited to this. Some of our participants discuss and important and transformative effect on their personal identity or perception of math education research. Thus we find that while emerging education researchers may struggle with theory, the process of learning about theory and attempting to overcome that barrier can be transformative.