C×N: Investigating the Creative Proving Process Using Neuroscience Methods

With this research, we seek to find theoretical constructs that correlate with participants' neural activity that occurs as they are presented slides of mathematical proofs. We first asked three graduate student participants to complete two graduate level proofs (one each of abstract algebra and real analysis) using a LiveScribe pen. We then generated slides of their written work and researcher-generated proofs that we used during electroencephalography (EEG) trials. Having coded the slides along 22 theoretical categories, we used step-wise model selection to determine suitable models for variance in neural activity. Preliminary results indicate that the best code-based models at a given instant can account for between 25 and 50 percent of the variance in electrical activity near the EEG electrode for that model when participants observe their own proofs and between 33 and 75 percent during researchers' proofs.

Key words: Proof, Electroencephalography, Insight, Mathematical Creativity

Throughout the mathematics education literature, researchers and mathematicians reference moments of insight during problem solving (e.g., Burton, 1999; Hadamard, 1945), which are also described as AHA moments (e.g., Liljedahl, 2004). And, although this notion is a common colloquialism among mathematicians and mathematics educators, little empirical research has focused on evidence beyond self-reported accounts of individuals' moments of insight as they develop formal proofs (Savic, 2015). To investigate and move beyond the self-reported moments of insights, we draw on neuroscience methodologies to provide hard data for exploring insight during proof. Through an experimental methodology using electroencephalography (EEG), coupled with an extended taxonomy created for local proof comprehension (Savic, 2011), we aim to identify and explore neural activity related to proof. Specifically, we seek to identify which theoretical constructs coded for a written proof might be used to model variance in participants' neural activities as they read their own and others' proofs. Ultimately, we are motivated by a broader investigation exploring the possibility of identifying neurological evidence of theoretical constructs related to proving, specifically whether we can identify neural correlates to moments of insight, an aspect of the creative process (Wallas, 1926).

Background Literature/Theoretical Framework

Wallas (1926) outlined four stages of creativity: preparation, incubation, illumination, and verification. Subsequent literature has drawn on these categories and used them to explore the creative process, beginning with mathematicians' reflecting on their own creativity (e.g., Hadamard, 1945; Poincare, 1946; Borwein, Liljedahl, & Zhai, 2014) and moving towards qualitative research relying on participants' subjective recollection of moments of insight (e.g., Burton, 1999; Liljedahl, 2004; 2013). However, Liljedahl (2004) conceded that there might be more that can be done to investigate insight, admitting, "Upon reflection, I now see that the clinical interview is not at all conducive to the fostering of such phenomena [insight]..." (p. 49). With this research, we aim to provide an additional approach exploring moments of insight.

Mathematicians and mathematics educators alike discuss moments during proof production in which a prover gains a clearer sense of why the conjecture they have set out to prove is true (e.g., Burton, 1999; Raman, 2003). Although researchers draw on varying perspectives and language to describe such moments and the psychological processes involved, these moments seem to occur during which the prover develops a sense of a "key idea" of the proof. We use

Raman's (2003) definition of key idea as something that "gives a sense of understanding and conviction. Key ideas show why a particular claim is true" (p. 323). Liljedahl (2004) stated that "At the moment of insight, in the flash of understanding when everything seems to make sense and the answer is laid bare before you, you know it, and you call out – AHA!, I GOT IT!" (p. 1).

In the neuroscience literature, there have been many studies dedicated to insight during problem solving (e.g., Bowden & Jung-Beeman, 2003). Kounios et al. (2006) found that "greater neural activity was observed for insight than for non-insight preparation in bilateral temporal cortex (left more than right, in both experiments)" (p. 887). Finally, insight has been correlated with phenomena such as P300 "positive deflection occurring 300 ms after stimulus presentation," and N200 - "negative deflection 200 ms post stimulus onset" (Dietrich and Kanso, 2010, p. 824). However, the prompts used in these psychology experiments are often remote association tasks, in which three words are given (e.g., electric, high, and wheel) and the participant is supposed to come up with a fourth word (e.g., chair) that can form an association among those three words (i.e., by being able to modify each word). Such tasks, while perhaps generating a moment of insight, may not afford the incubation time that people might need while proving theorems (Savic, 2015).

Researchers from several neuroscience domains – including memory, vision, and motor control – have demonstrated strong evidence supporting the notion that by either remembering or imagining an event or action individuals tend to generate neural activity that is similar to the activity generated during the actual event or action (e.g., Farah, 1988; Stavrinou et al., 2007). Our central hypothesis is that when subjects re-experience their proofs (i.e., see their proofs during the experiment), they might experience similar thinking to that which occurred during the moment of insight that produced a critical step for completing the proof. So, it stands to reason that the neural activity generated during this "re-living" experience will be similar to the brain activity generated during the production of the proof.

Methods

Our experimental methodology combines both theoretical and neuroscience components. We used a modified taxonomy for local proof comprehension (Savic, 2011) to generate codes for each slide of a slideshow that each participant watched during the EEG trial. In the first phase (days 1-2), three graduate mathematics majors (Marshall, Francis, and Buzz) were provided with a Livescribe pen to record his or her proving process, which we call "original" proofs. This pen is able to record the participant's written work and in real-time, allowing each pen stroke to be timestamped (to the second) and synchronized with the participant's verbal utterances during their proof. The pen also allows the participants' work to be saved as digital images. They were provided with two theorem statements (Figure 1), one in abstract algebra and the other in real analysis. We selected theorems that a senior-level abstract algebra and real analysis should reasonably be expected to prove.

Task 1: Prove that no group is the union of two of its proper subgroups.
Task 2: Prove that, if $a \in \mathbb{R}$ and $f\colon \mathbb{R} \to \mathbb{R}$ and $g\colon \mathbb{R} \to \mathbb{R}$ are functions continuous at a, then $fg\colon \mathbb{R} \to \mathbb{R}$ is continuous at a. [Here $(fg)(x) = f(x)g(x)$. Note that $fg \neq f \circ g$.]

Figure 1: Theorem statements to be proven by participants

In the second phase (days 3-4), each participant's digitized proofs were chunked and converted into uniform-sized digital images (700x700 pixels). In addition to the two proofs created by the participants, we generated two pre-written proofs of related, but different, conjectures which we call "canned" proofs. We generated these proofs to be concise, mathematically correct (not misleading) proofs of proximal difficulty as the assigned proofs for the participants to allow us to compare brain activity for a proof generated by the participant with one that was not.

In the third phase (day 5), participants came to the laboratory and were fitted with a 128-channel net, where the EEG data was obtained. Participants were instructed to watch the slideshows of the "canned" proofs and try to follow along with the argument as they would in a lecture or reading a text. For the "original" proofs, we instructed them to try and remember their thought processes as they generated each proof, focusing on their reasons for writing the content of each slide. The canned abstract algebra proof was shown first in its entirety, followed by their own abstract algebra proof, and then the canned real analysis proof, and finally their own real analysis proof. Each slide was shown for three seconds with a subsequent one-second break to blink, and all four groups of slides were shown in one sitting. Following the presentation of the last proof, participants were interviewed, with the entire interview taking between 15 and 45 minutes. This semi-structured interview inquired about academic history (e.g., mathematics courses completed) and demographic data. We also asked each participant to explain what s/he thought it meant to be the key idea of a proof. We then provided each participant with paper copies of their own proofs and the researcher-generated canned proofs, and asked him or her to identify the most important part(s) of the proof.

Variables and analyses

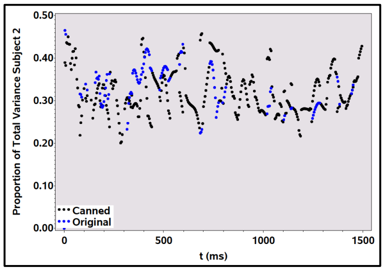
After data collection, at least two researchers from the team separately coded the collections of slides according to the modified proof comprehension taxonomy (Figure 2) and met to compare codes. Discrepancies between codes were resolved through discussion between the coders. We also generated an additional code of "student-reported potential for insight" (SRPFI) based on the slides corresponding to parts of the proof that the participant identified as important during the post-EEG interview. For the EEG data, for each time point (every 4 milliseconds between 0 ms and 1500 ms) in every slide across the four trials, we regressed the measured amplitude of the EEG electrode with the theoretically coded variables, choosing the final model through stepwise regression. In this process, we chose the electrode with greatest predicted total proportion of the variance (R^2) explained using the fewest coded variables. Here, variance in a given electrode is identified as the difference across all slides between the voltage of measured electrical activity by that electrode for a fixed time elapsed after first seeing each slide. We selected these models at each time point to generate a spatial and temporal description of the evolution of the modeled neural activity induced by the slides of the proofs.

Assumption (A), Contradiction statement (CONT), Delimiter (D), Exterior reference (ER), Interior reference (IR), Relabelling (REL), Statement of intent (SI), Similarity in a proof (SIM), Algebra (ALG), Conclusion statement (C), Definition of (DEF), Formal logic (FL), Use of exterior reference (UE), Use of interior reference (UI).

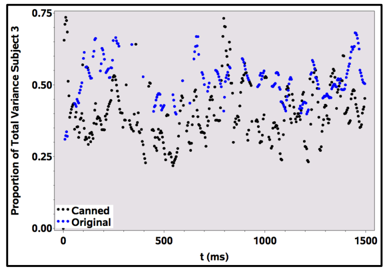
Figure 2: List of Codes from modified local proof comprehension taxonomy (Savic, 2011)

Results

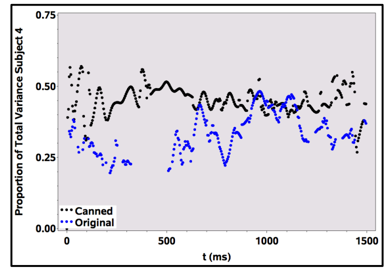
Our preliminary analysis indicates that the theoretical codes are able to account for between 20% and 75% of variance (y-axis in Figure 3) in modeled neural activity, though the codes accounted for around 35% to 50% of variance in neural activity for the majority of the time (x-axis). As one might infer from Figure 3, the current data do not show a conclusive difference between the potential for the theoretical codes to explain (or predict) variance between the canned and original proofs.



(a) Marshall



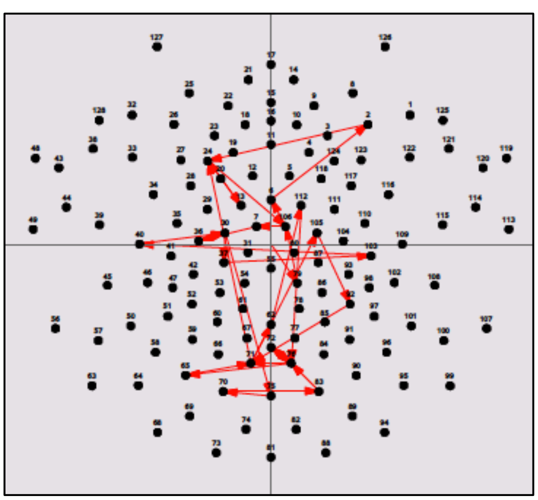
(b) Francis



(c) Buzz

Figure 3: Graphs of the proportion of total variance predicted by the model of best fit along time

We have also generated maps of the progression over time of the EEG electrodes with most variance explained (Figure 4). The figures below display the placement of the 128 nodes from the neural net. The highest nodes in quadrants 1 and 2 are located under the eyes. The origin corresponds with the top-most, center part of the participant's head, quadrant I corresponds with the right frontal lobe; quadrant II corresponds with the left frontal lobe, quadrant III corresponds with the left occipital lobe, and quadrant IV corresponds with the right occipital lobe. Initial comparisons between the maps of the participants' responses to the researcher-generated "canned" proofs and their responses to their own proofs indicate that the participants' react to the two types of proofs differently. For instance, the modeled electrodes during Marshall's reaction to the "canned" proof (Figure 4a) initially concentrated in and around the parietal lobe and shift to the temporal and frontal lobes. This implies that Marshall's initial reaction to a slide from the "canned" proofs tended to rely initially on visual processing before shifting to critical processing areas in the brain.


(a)

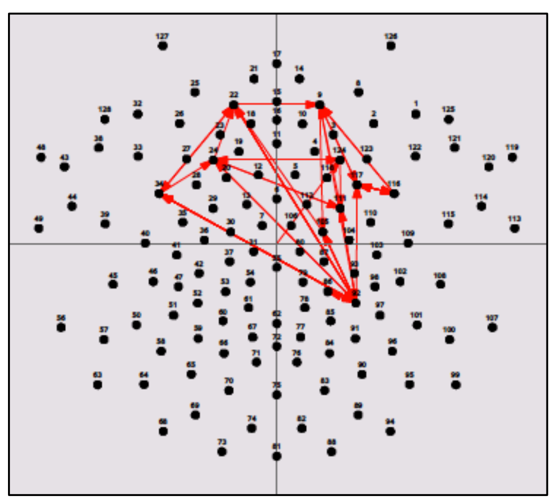
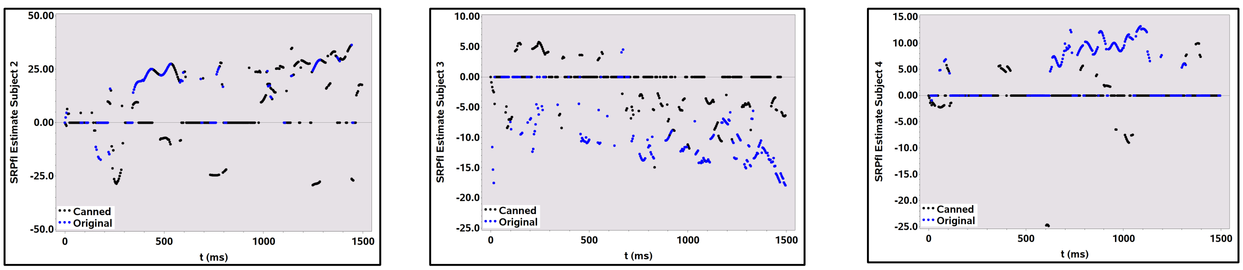

(b)

Figure 4: Progression of neural activity with best model fit for Marshall's canned proofs (a) and

original proofs (b).

In contrast, when Marshall was presented with "original" slides, the neural activity modeled in our regression takes place almost entirely in the temporal and frontal lobes (Figure 4b). Since his dominant electrodes were not in the visual processing area (occipital lobe), this supports the notion that Marshall is relying on his prior experiences as he observes the "original" slides. In other words, Marshall is likely engaged in some level of "re-living" his own proof. Finally, we see bilateral movement between left and right hemispheres on both 4a and 4b, which is "essential for complex mathematical reasoning" (Dehaene et al., 1999; cited in Desco et al., 2011, p. 282).

We found that all three participants had the code SRPFI (student-reported potential for insight) included in many of the regression models, seen in Figure 5, where the y-axis is microvolts squared and the x-axis is time. This indicates a potential link between student-reported insight and neural activity. However, due to the preliminary nature of the findings and sample size, we only hope to expand our research on that link.



(a) Marshall (b) Francis (c) Buzz

Figure 5: Graphs of the microvolts^2 of variance influenced by SRPFI

Future Research

We hope to extend our current findings by (1) exploring interpretations of our current data and the theoretical implications this might have on the codes we use to model neural activity, (2) collecting data with more participants, and (3) exploring different neural data collection techniques (e.g., alternative brain-computer interfaces such as fNIRS – functional near-infared spectroscopy). We expect that further analysis of the data collected to this point will help inform future work by raising questions of which theoretical constructs are able to better predict variance in neural activity, in turn narrowing our focus with respect to the existing theoretical codes and also informing our selection of potential codes to use in the future. Can we, with only coding, predict when a person has an insight in his or her proving process? Finally, since the codes considered local proof comprehension, we plan on using other coding schemes for holistic proof comprehension.

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