# SOURCES OF EVOLUTION OF UNIVERSITY STUDENTS' VIEWS ON MATHEMATICAL CREATIVITY

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In a world that is in increasing demand for creativity, mathematics courses and programs need to shift from more routine and computational to more creative and problem-solving focused. We present preliminary results of a qualitative research study in which we examined students' perceptions of mathematical creativity in an introduction-to-proofs course. We conducted interviews with students as well as collected their reflection assignments at the end of the semester. Using a definition of creativity from a relativistic perspective, we analyzed interview data to describe how students' perspectives of mathematical creativity evolved throughout the semester and the sources of those shifts. Students shifted from previously not seeing themselves, others, or mathematics as creative, to believing they are creative. The sources found in the data are related to content and course design.

Keywords: University Mathematics, Creativity, Affect, Emotion, Beliefs, and Attitudes

# Introduction

Curriculum-standard documents, both in the United States and internationally, mention creativity as an important skill when learning mathematics (Askew, 2013). Additionally, creativity has become one of the most sought-after skills for academia and industry employers (World Economic Forum, 2016). While the mathematical creativity literature at the K-12 level is well-developed, there remain few studies at the undergraduate level and fewer still that investigate students' beliefs about creativity and its role in mathematics. In this qualitative study, we explored students' perceptions of mathematical creativity and how they evolved over the semester of an introduction-to-proofs course. Furthermore, we examine the sources of these shifts as evidenced by the students' own words.

# **Theoretical Perspective**

As with many of our research projects on mathematical creativity (Tang et al., 2015; Savic, Karakok, Tang, El Turkey, & Naccarato, 2017), this study uses a *developmental* perspective of creativity (Kozbelt, Beghetto & Runco, 2010). This theoretical lens contends that creativity develops over time and emphasizes the role of the environment in the development of creativity. Such an environment should provide students authentic mathematical tasks and opportunities to interact with others (Sriraman, 2005).

We operationalize mathematical creativity as "a process of offering new solutions or insights that are unexpected for the student, with respect to their mathematical background or the problems [they've] seen before" (Savić et al., 2017; p.1419). This definition focuses on the *process* (Pelczer & Rodriguez, 2011) of creation, rather than the *product* that is created at the end of a process (Runco & Jaeger, 2012).

## **Background Literature**

Moore-Russo and Demler (2018) examined the perceptions of U.S. faculty and staff participants from gifted mathematics programs and found that, through counts of coding using several creativity frameworks, mathematical creativity in education was more of a process than "a subjective experience" (p.23). This particular orientation allows us to keep a dynamic view rather than a static one to capture nuances in the individual's thinking. Furthermore, our definition takes a *relativistic* perspective—creativity relative to the student—in contrast to absolute creativity for the field of mathematics (Leikin, 2009). For example, Levenson (2013), using a similar viewpoint, focused on the discussion of ideas put forth by individual students and how these ideas helped in developing a product of collective mathematical creativity in fifth- and sixth-grade mathematics classrooms. Levenson also emphasized the teachers' roles in facilitating these discussions.

While there is literature on mathematicians' and mathematics instructors' perceptions on mathematical creativity (Borwein, Liljedahl & Zhai, 2014; Sriraman, 2009), research on students' perceptions on mathematical creativity as well as classrooms that impact these perceptions has received less attention. In one of our earlier studies, we examined university students' and mathematicians' definitions of mathematical creativity using three process categories: taking risks, making connections, and creating ideas (Tang, El Turkey, Savić, & Karakok, 2015). We found that students rarely associated making connections using different mathematical content with creativity compared to mathematicians (9% of students' responses compared to 38% of mathematicians' responses). This study alerted us to think about explicitly valuing and discussing the processes that are deemed to be important in developing mathematical creativity (El Turkey et al., 2018). In this paper, we explore the following research question: In what ways do students' views on creativity evolve in an introduction-to-proofs course which explicitly valued mathematical creativity?

#### Methods

Data were collected in an introduction-to-proofs course at a small liberal arts college in the Southwestern United States. This course was taught using an inquiry-based learning (IBL) pedagogy (Laursen et al., 2014), where students often worked on proofs in small groups and gave presentations to the class on proofs constructed both in class and for homework. The instructor explicitly valued creativity by making use of the Creativity-in-Progress Rubric (CPR) on Proving (Savić et al., 2017; El Turkey et al., 2018), a formative assessment tool developed by the authors that students can use to persevere in proving and encourage creative processes. The rubric has two main categories: making connections and taking risks (see Author, 2017 for a more detailed discussion of the CPR on Proving). The instructor gave assignments and exam questions where students had to use the rubric to assess their own or other's work.

At the end of the semester, 4 female and 3 male students agreed to be interviewed and participated in 60 to 90-minute semi-structured interviews. During the interview, students were asked to describe the course, discuss their views on creativity, and discuss the use of the CPR in the course. As part of a larger study, interviews were coded using hypothesis coding (Saldaña, 2013) with five categories, one of which being creativity. This is the coding category we focus on for this report. Three of the seven participants' transcripts were coded separately by the first and second author with 97% agreement. Because of this high degree of inter-rater reliability, the remaining transcripts were coded by only the first author.

## Results

From three of the students interviewed (all of whom identified as female), an explicit shift in the way they thought about creativity or how they viewed themselves as creative people was reported. The students that reported an evolution in perspective on creativity were able to ascribe this to one of two sources: mathematical content and course design. In what follows, we show a sampling of student quotes where they indicate a shift in perspective and ascribe a reason to this change.

For instance, Stephanie (all names reported are self-chosen pseudonyms) spoke about content with respect to *learning new tools* to work with. That is, she feels that having a larger mathematical toolbox allows one to be more creative when proving or problem solving.

I think I started to look at creativity a little bit different through this course...Prior to this it's been all very applied mathematics...So before, just using the trig equations to solve geometry was creative for me. Whereas now, this has just opened up a whole new door of opportunities for it because I can solve a proof using a contradiction, while somebody else used a contrapositive and somebody else used a direct proof and somebody else used induction, and we all do it completely different.

Whereas, Olivia attributed her shift to the social structure of the course. As the course included collaboration and presentation, Olivia reported that the environment was conducive for growth and students were able to see each other's creativity and began to feel more creative as the semester progressed.

We kind of all went in with kind of not really feeling confident in our abilities to be creative, so it was really interesting to see students that were quiet, reserved early on like show their work later in the semester and they had done something like totally cool and amazing...So, I feel you know their ability, like their confidence levels went up and I could say that's true of me as well. So, I wanna say that it's, you know it wasn't that like all the creative people took this course because I didn't consider myself creative and I took the course, and I would say that that's probably true of other students as well.

In a later part of her interview, Stephanie echoed Olivia's comment almost exactly with her assessment of the course culture and its contribution to everyone's creativity.

At the beginning of the semester, I think a lot of people in that class were very shy and quiet, and so it was kind of hard to judge where their creativity was because they weren't sharing it as much. Um, by the end of the course you had everybody speaking, you had everybody giving their opinions and how to work on things together, and you saw everyone grow. You saw everyone coming up with their own tools and tricks. And everyone was posing questions, not just the few of us that were outspoken to begin with. So, you definitely saw growth in the class, um not only with the shyness but with the creativity and coming up with their own ideas to change things and make them better.

The IBL practices of the course required students to present their work to each other. The instructor also especially encouraged multiple presentations on the same problem if different students approached the problem using different methods. Two of the interviewees spoke directly to this aspect of the course design as contributing to their own creativity. That is, this shift seems to be a result of *seeing others' work as creative and reflecting it back on themselves*. For instance, Peyton said:

I really, I really did not feel like I was being creative at all throughout the course. It really was just things in my head, it makes sense that led to a conclusion that made sense. But, considering that I thought other people were exceptionally creative, I kind of thought that maybe they though that about me too.

In fact, Peyton had perhaps the starkest change in her beliefs on mathematical creativity and in seeing herself as a creative person. The following excerpt shows that Peyton started the semester believing that mathematics was not a creative subject and ended with a completely opposite viewpoint.

- Interviewer: And in your reflections you said...'I think I am on the spectrum that generally believes that, believes there is no need for creativity in mathematics. That's been a key reason why I enjoy math. I know, I know if I get the answer then I have done it correct. There is a set process and if I learn the process then...I'll be successful'. So, do you wanna comment on that part?
- Peyton: I...should have made that more in the past tense, because I believed that prior to taking this course...There has been, you can figure out problems and it's creative in the sense that you can figure out how, where you wanna start with the problem. But I like being able to know that if I am doing it correctly, the process correctly, then I will get to the answer... I enjoy knowing when I'm gonna do something correctly as opposed to just spending a lot of time and then not even knowing if it's gonna yield good results. But this course changed that quite a bit, because there really was no assurance that anything would be correct, but it still... required me to use different thought processes to get to a result hoping for the best, which was stressful to say the least, but still, it was fun.

# Discussion

These three females explicitly acknowledged that their previous perceptions of not seeing themselves, others or mathematics as creative shifted to thinking they or mathematics are creative. We found two main sources of these shifts a) *content* - having more mathematical tools to work with, b) *course design* - developing a mathematical community that allows students to see each other's creative work with opportunities to reflect and connect back to their own work. Thus, for these students, content and course design seem to be important sources in shifting students' perceptions of themselves, others, or mathematics as creative.

Furthermore, although Stephanie does not explicitly mention the CPR on Proving, she mentions two of the subcategories "Tools and Tricks" and "Posing Questions". By using the CPR on Proving, it is evident that this particular instructor's course design and teacher actions aimed to explicitly value and foster students' mathematical creativity. This facilitated the evolution of students' perspectives on mathematical creativity. The connection between course design, teachers' actions, and changing students' perspectives on mathematical creativity requires additional exploration and our future work aims to examine this connection in detail and catalog specific *creativity-fostering teacher actions*. In particular, we wish to determine not only which teacher actions are more fruitful to afford such changes, but also what other course design features can contribute to shifts in student appreciation of mathematical creativity and fostering of creative behavior in the classroom.

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