AN INITIAL INVESTIGATION INTO TEACHER ACTIONS THAT SPECIFICALLY FOSTER MATHEMATICAL CREATIVITY

Emily Cilli-Turner[1], Milos Savic[2], Houssein El Turkey[3], Gulden Karakok[4]
[1]University of La Verne, [2]University of Oklahoma, [3]University of New Haven, [4]University of Northern Colorado, USA

***Abstract**: While mathematicians and mathematics educators agree that students should be exposed to the creativity inherent in mathematics, there still is a need for further research showing how this can be done at the tertiary level mathematics. This report uses empirical evidence in conjunction with Sriraman's Five Principles for maximizing creativity framework to explicate teaching practices that can foster mathematical creativity in the classroom. The report provides a practical guide for mathematics teachers who would like to value and nurture creative mindsets in their students.*

Key words: teaching practices, mathematical creativity, tertiary-level

INTRODUCTION

Mathematical creativity seems to be an important part of mathematics (Hadamard, 1945), and more recently, mathematics education (Schumacher & Siegel, 2015). There are researchers that have studied pedagogical actions of fostering mathematical creativity at the K-12 level (e.g., Levenson, 2011) and in the tertiary level (e.g., Zazkis & Holton, 2009). We believe a more in-depth investigation, a theoretical backing (Sriraman, 2005), and verification will add to our understanding of ways to foster mathematical creativity.

In this report, we take the five principles conjectured by Sriraman (2005) and expand them to twenty actionable items. Then, using data from an inquiry-based learning tertiary classroom on introducing proofs, we offer student testimonials that they were creative and why they felt creative. These explanations are analyzed using the twenty teacher actions. We conclude the proposal with new possibilities for future research.

THEORETICAL FRAMEWORK

We view mathematical creativity as a process of offering new solutions or ideas that are unexpected for the student, with respect to their mathematics background or the problems they have seen before (Savic et al., 2017a). A heavy influence of our definition came from Liljedahl and Sriraman's (2006) discussion on mathematical creativity and its constructs in the classroom. Focus on the creative "process" is one of four major theoretical perspectives in researching creativity: the viewpoint of the *person*, the *product* that arises, the *process* by which that product is created, and the *press* or the response that the product elicits from others (Rhodes, 1961). It is difficult to define what "new solutions or ideas" and "unexpected" are; therefore, with our definition, the students designate what "new" is, grounding originality within "the student" and their background. Vygotsky (1984; as cited by Leikin, 2009) stated that there is significance in both *relative* (how we employ the definition) and *absolute* creativity, with absolute being discoveries at a global level. This is similar to the Big-C and little-c creativity discussed by many researchers (e.g., Beghetto & Kaufman, 2007). Focus on mathematical creativity instead of creativity in general relies on the notion that there are significant differences of creativity between domains (Baer, 1998).

This student-centered, process-oriented perspective yields a central question: **how does one foster such mathematical creativity?** Zazkis and Holton (2009, pp. 359-360) described both problems and creativity-fostering actions, including multiple solution tasks (Leikin, 2009), learner-generated examples, open-ended problems (Zaslavsky, 1995), and creating new mathematical definitions. We add to this literature by demonstrating creativity-fostering actions in one classroom derived from Sriraman's principles below.

Sriraman's Five Principles for Maximizing Creativity

Sriraman (2005) conjectured five principles for maximizing creativity in mathematics: gestalt, aesthetic, free-market, scholarly, and uncertainty. We say "conjectured" since these were recommendations according to Sriraman and have been minimally investigated in the classroom (Savic et al., 2017b).

The *Gestalt principle* is based off of Gestalt psychology and Wallas' four-stage creative problem-solving process: preparation, incubation, illumination, and verification (Wallas, 1926; Hadamard, 1945). This principle requires that instructors allow students "to engage in suitably challenging problems over a protracted time period, thereby creating the opportunities for the discovery of an insight and to experience the euphoria of the "Aha!" moment" (p. 27). The *aesthetic principle* looks at the beauty of a mathematical process or solution. Characteristics such as elegance, efficiency, atypical, and combination of disparate ideas are part of the aesthetic principle. The *free market principle* revolves around taking risks when presenting a solution. Sriraman explains: "Professional mathematicians… take a huge risk when they announce a proof…The implication… for the classroom is that teachers should encourage students to take risks…allowing them to gain experience at defending their ideas upon scrutiny from their peers" (p. 28). The *scholarly principle* looks at creating an environment where "[teachers] should be flexible and open to alternative student approaches to problems… nurture a classroom environment in which students are encouraged to debate and question the validity of both the teachers', as well as other students', approaches to problems… (p. 28). The *uncertainty principle* is based on the idea that mathematics as a discipline involves uncertainty and we should expose our students to that concept by appealing to the history of mathematics and showing that many problems to years to solve.

TEACHER ACTIONS THAT FOSTER CREATIVITY

The data presented in this report were collected in Spring 2016 from students in an introduction to proof course at an institution in the Southwestern United States. To explicitly value creativity in the classroom, the instructor, Dr. Eme, used the Creativity-in-Progress Rubric (CPR) on Proving (Savic et al., 2017) while implementing an Inquiry-Based Learning (IBL) teaching pedagogy. Fourteen students were invited to participate in interviews at the end of the semester and seven participated. The interviews were conducted by a member of the research group and were from 15-75 minutes in length.

Two members of the research team coded the interviews separately and then met to discuss their codes and consensus on codes was reached. For the majority of the coding process, the two members used leading questions based on the 5 principles to code the interviews. For example, to code for the Free Market principle, we looked through the interviews for answers to "Did the course or instructor's actions/teaching promote students to take risks while presenting solutions? Did the course or instructor's actions/teaching create a safe environment for students to take risks?'' The following student quote was coded under the Free Market principle as it answered these questions:

> I think really the structure of the course is what helped to expand on my mathematical creativity when I thought I didn't have any. So, um, and you know the structure of the course meaning, you know the group discussions, the group talks, um the presentations were a pretty big deal.

Based on student answers to the question "Did you feel creative in this course?" we found that students reported feeling very creative in the course as well as recognizing and valuing the creativity of other students. For example, two of the interviewees' responses were:

> In regard to mathematics, I think I am on the spectrum that generally believes there's no need for creativity in mathematics. That's been a key reason why I enjoy math. I know if I get the answer then I have done it correct. There's a set process and if I learn the process then I will be successful. However, this class especially has proven to me how untrue that belief is.

> …working with…trapezoidal numbers, and once we saw the different representation of consecutive numbers, you know minus 1 and plus 1 versus plus 1, plus 2, which was all, the entire class' first initial connection was, you know plus 1 and plus 2. When he flipped it to the other side, everyone was just 'Wowww! That's so amazing!' And then but, then we went on and worked with trapezoidal numbers a little bit more, and everybody's making that connection.

This led to our research question: What specific teacher actions that contributed to the feeling of creativity amongst the participants in the classroom community? Dr. Eme seemed to use various teacher moves in her classroom, and we have triangulated moves with instructor journals and interview data as well as students' interviews.

One student stated that Dr. Eme valued their contributions without passing judgement.

> so I think when she … gave us like that reflection of like what it means to be creative, we kind of, she kind of just like told us like 'No proof is gonna be exactly the same. Like none of your proofs are actually gonna be the same as each other and you guys are all gonna come up with different ideas'. And she kind of like helped us, like she never like hindered those ideas. She was like 'Oh, well maybe it can work like this. Maybe it can work like that. You just have to like see'.

This teacher action aligns with the Scholarly principle from Sriraman as students were encouraged to present their work to the instructor or other students and other students were encouraged to build on that work.

Another student spoke to the fact that Dr. Eme allowed for multiple attempts on problems.

> I think also the feedback that Dr. [Eme] would give us on our homework. Cause we would turn it in and we would be able to have multiple submissions of our homework to make sure that we would get the proof right.

Allowing students to turn in proofs more than once encourages students to take risks as outlined in Sriraman's Free Market principle. This student is also telling us that the instructor was allowing students to try an approach and fail without penalty. Interestingly, this quote also reflects the instructor implementing the Gestalt principle by allowing freedom of time and movement, giving students a chance to reach that AHA moment.

Another interviewee spoke to the instructor's style of giving guidance, but not answers to student questions.

> I think a lot of it was the way the class was structured and Professor [Eme] gets a lot of credit for that. She very much threw us in there and said 'sink or swim'. And you know it was 'I'm here if you need a little guidance but you're never gonna get an answer from me, so don't even bother asking for an answer; you know it's not about the answer it's about the process.

Dr. Eme allowed students to experience the difficulty and uncertainty involved in doing mathematics. Therefore, she exposed students to the authentic practice of being a mathematician, where a path to solving a problem may not be clearly defined or may not even exist.

The next student quote demonstrated the instructor implementing the Aesthetic principle and encouraging students to see the beauty in mathematics.

> There's one guy in particular who had a way of coming up with these tricks that just made proofs very efficient. Instead of having ten lines, he would have three and it would be fully proved. And it was really neat…..It was wonderful watching his work.

Through encouraging students to see each other's work (which could be considered an implementation of the Scholarly principle), students would judge other students work, both for correctness and form, but also for aesthetic appeal. This process is expanded upon by another student interviewee reflecting on the work of a student she labeled as particularly creative.

> So his creative moment, I could then use to expand on and do something a little different with to have my own creative moment. And then I could show that to the class and then you know somebody else in class could pick that up and manipulate it for a different proof and do other things with it. And so, we were doing these things that are not the road most travelled I guess and then … those become an integral part of the road we are traveling together, and yet each time we're changing it to be what we need it to be and expanding on it and having our own creative moments, based on a creative moment that somebody else had before us.

Teacher Actions Extracted from Sriraman's Five Principles

The above quotes speak to teacher moves that the students were experiencing in the classroom that encouraged their creativity. These teacher actions have a high alignment with Sriraman's Five Principles. In fact, based on our attempts to explicate the actions encapsulated in Sriraman's (2005) Five Principles, we enumerate below twenty specific teacher actions that can maximize student creativity. Several of these teacher actions are the same as we showed in the last section based on the student quotes, however we discuss additional actions here that were not observed in this classroom yet are highlighted in the Five Principles. The teacher actions outlined here are more detailed than the Five Principles themselves and displaying them in this way makes them accessible to be implemented by any instructor wanting to foster creativity in their classroom.

The Gestalt principle contains three specific teacher actions within it. Following this principle, a teacher should:

- allow for freedom of time and movement;
- discuss explicitly that time, effort, and energy are needed to solve problems;
- assign challenging problems and tasks.

Allowing for freedom of time and movement may incorporate classroom practices such as giving flexible due dates to allow time to really work through a problem, allowing revisions of problems, and encouraging different approaches to problems.

Four relevant teacher actions are covered by the aesthetic principle. When applicable to the classroom situation, a teacher should:

- point out the elegance/novelty/beauty of certain solutions/approaches;
- point out connections between disparate ideas in problem solving;
- point out any atypical thinking/solutions;
- point out simple solutions to complex problems.

Our study shows that the free market and scholarly principles have a lot of overlap, thus we present nine teacher actions that relate to one or both of these principles. To enact the free market and scholarly principles, a teacher should:

- encourage students to present their solutions and approaches;
- encourage students to defend their solutions and approaches;
- value students' contributions;
- not penalize students for trying a different approach and failing;
- encourage students to debate and discuss the teacher's approaches and the other students' approaches/presentations;
- elaborate on how these discussions contribute to the process of knowledge building;
- point out when a student builds on the work of another student;
- encourage students to make generalizations;
- allow students to problem pose.

Finally, there are four teacher actions embodied in the uncertainty principle. For this principle, a teacher should:

- point out the difficulty and uncertainty of doing mathematics when students are working on challenging tasks;
- provide affective support to students when they experience frustrations;
- encourage perseverance;
- expose students periodically to examples from history to explain that certain concepts took years/centuries to develop.

DISCUSSION

The student interview data in this study provided evidence that students felt creative in this classroom and were also able to value creativity in other students. Additionally, students were able to identify specific practices of the instructor that contributed to these feelings. These teacher practices along with the ones extracted from Sriraman's Five Principles provide a robust framework for a classroom that fosters creativity. Making these teaching practices explicit from the Five Principles allows instructors that want to encourage creativity in their classroom to "pick from the menu" of practices and implement them.

These twenty teacher actions also present several open questions that need future research. The classroom discussed in this report did not incorporate all twenty teacher actions, yet still was successful in fostering student creativity. Therefore, which teacher actions are the most or least important in promoting a creativity-focused learning environment? Is there some minimal spanning set of teacher actions? That is, is there a least number of practices that one could implement and still see similar results to the classroom presented here?

References

Baer, J. (1998). The case for domain specificity of creativity. *Creativity Research Journal, 11*(2), 173–177.

Beghetto, R. A., & Kaufman, J. C. (2007). Toward a broader conception of creativity: A case for "mini-c" creativity. *Psychology of Aesthetics, Creativity, and the Arts, 1*(2), 73-79.

Hadamard, J. W. (1945). *Essay on the psychology of invention in the mathematical field*. Princeton, NJ: Princeton University Press.

Leikin, R. (2009). Exploring mathematical creativity using multiple solution tasks. In R. Leikin, A. Berman & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 129–145). Haifa: Sense Publishers.

Levenson, E. (2011). Exploring collective mathematical creativity in elementary school. *The Journal of Creative Behavior*, *45*(3), 215-234.

Liljedahl, P., & Sriraman, B. (2006). Musings on mathematical creativity. *For the Learning of Mathematics*, 26(1), 20–23.

Rhodes, M. (1961). An analysis of creativity. *The Phi Delta Kappan, 42*(7), 305-310.

Savic, M., Karakok, G., Tang, G. El Turkey, H., & Naccarato E. (2017a). Formative assessment of creativity in undergraduate Mathematics: Using a Creativity-in-Progress Rubric (CPR) on Proving. In R. Leikin & B. Sriraman (Eds.), Creativity and Giftedness (pp. 23-46). Springer International Publishing.

Savic, M., El Turkey, H., Tang, G., Karakok, G., Cilli-Turner, E., Plaxco, D. & Omar, M. (2017b). Pedagogical Practices for Fostering Mathematical Creativity in Tertiary-Level Proof-Based Courses. In D. Pitta-Pantazi (Ed.), Proceedings of the 10th Biannual Conference on Mathematical Creativity and Giftedness (pp. 130-135). Nicosia, Cyprus.

Schumacher, C. S., & Siegel, M. J. (2015). *2015 CUPM Curriculum Guide to Majors in the Mathematical Sciences*. Washington, DC: Mathematical Association of America.

Sriraman, B. (2005). Are giftedness and creativity synonyms in mathematics? *Prufrock Journal, 17*(1), 20-36.

Sriraman, B., & English, L. (2004). Combinatorial mathematics: Research into practice. *The Mathematics Teacher*, *98*, 182–191.

Vygotsky, L. S. (1984). Imagination and creativity in adolescent. In D. B. Elkonin, *Vol 4: Child Psychology. The Collected Works of L. S. Vygotsky* (pp. 199-219). Moscow, SSSR: Pedagogika.

Wallas, G. (1926). *The art of thought*. New York: Harcourt Brace.

Zazkis, R., & Holton, D. (2009). Snapshots of creativity in undergraduate mathematics education. In R. Leikin, A. Berman, & B. Koichu, *Creativity in mathematics and the education of gifted students* (pp. 345-365). Rotterdam, the Netherlands: Sense.