TERTIARY STUDENTS’ Changing Views on Mathematical Creativity

Emily Cilli-Turner\textsuperscript{a}, Milos Savic\textsuperscript{b}, Gulden Karakok\textsuperscript{c}, Houssein El Turkey\textsuperscript{d}, Gail Tang\textsuperscript{a}

Presenting Author: Emily Cilli-Turner (ecilli-turner@laverne.edu)
\textsuperscript{a}Department of Mathematics, University of La Verne, La Verne, CA, USA
\textsuperscript{b}Department of Mathematics, University of Oklahoma, Norman, OK, USA
\textsuperscript{c}School of Mathematical Sciences, University of Northern Colorado, Greeley, CO, USA
\textsuperscript{d}Department of Mathematics, University of New Haven, New Haven, CT, USA

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ABSTRACT

Tertiary students represent the near-future leaders and employees of science and mathematics. In a world that is in increasing demand for creativity, our mathematics courses and programs need to shift from more routine and computational to more creative and problem-solving focused. In this paper, we present preliminary results of a qualitative research study in which we examined students’ perceptions of mathematical creativity in a transition to proof course. In our investigation, we conducted interviews with students as well as collected their reflection assignments at the end of the semester. Using a definition of creativity from a relativistic perspective, we analysed interview data to describe students’ perspectives of mathematical creativity by the end of the semester and how their reported views evolved. Our findings indicate that undergraduate students have robust views of creativity and showed numerous shifts in how they felt about creativity or how they saw themselves as a creative person.

INTRODUCTION

Creativity has become one of the most sought-after skills for academia and industry employers (World Economic Forum, 2016). Additionally, the importance of creativity is highlighted in curriculum-standard documents internationally (Askew, 2013). Cropley (2015) summarized these points: “[t]eaching engineers (and other STEM disciplines) to think creatively is absolutely essential to a society’s ability to generate wealth, and as a result provide a stable, safe, healthy and productive environment for its citizens” (p.140). While difficult to define (Mann, 2006), mathematical creativity may even be more critical in science, technology, engineering, and mathematics (STEM), since mathematics is so prevalent and acts as a gatekeeper in STEM fields (Carnevale, Smith & Strohl, 2013). The number of studies examining students’ mathematical creativity at the tertiary level and how to enhance it is slowly growing, but compared to the number of studies at primary and secondary school mathematics level, it is still sparse.

To address this particular need we, as the creativity research group, have been conducting studies at the tertiary level mathematics courses (Karakok, Savic, Tang & El Turkey, 2015; Tang, El Turkey, Savic & Karakok, 2015; Savic, Karakok, Tang, El Turkey & Naccarato, 2017; El Turkey, Tang, Savic, Karakok, Cilli-Turner & Plaxco, 2018; Omar, Karakok, Savic & El Turkey, 2019). In this paper, we share preliminary results of a research study that we conducted in an introduction-to-proofs course. In this qualitative study, we explored students’ perceptions of mathematical creativity and how their perspectives evolved over the period of the course.

THEORETICAL PERSPECTIVE AND BACKGROUND LITERATURE

Our research projects on mathematical creativity can be grounded in the Developmental perspective of creativity (Kozbelt, Beghetto & Runco, 2010). The developmental perspective asserts that creativity develops over time and emphasizes the role of the environment in which students are provided authentic tasks and opportunities to interact with others.

We operationalize mathematical creativity as “a process of offering new solutions or insights that are unexpected for the student, with respect to their mathematical background or the problems they’ve seen before” (Savic et al., 2017; p.1419). Our focus in this definition is on the process (Pelczer & Rodriguez, 2011) of creation, rather than the product that is created at the end of a process (Runco & Jaeger, 2012). This particular orientation allows us to keep a dynamic view rather than a static one to capture nuances in the individual’s thinking. Furthermore, the definition takes a relativistic perspective—creativity relative to the student—in contrast to absolute creativity for the field of mathematics (Leikin, 2009; Beghetto & Kaufman, 2013).

The process and relativistic perspectives are particularly important in exploring how to enhance students’ mathematical creativity. For example, Levenson (2013), using a similar viewpoint, focused on the discussion of ideas put forth by individual students and how these ideas helped in developing a product of collective mathematical creativity in fifth and sixth grade mathematics classrooms. Levenson also emphasized the teachers’ roles in facilitating these discussions. Moore-Russo and Demleir (2018) examined the perceptions of U.S. faculty and staff participants from gifted mathematics programs and found that, through counts of coding using several creativity frameworks, mathematical creativity in education was more of a process than “a subjective experience” (p.23).

Nevertheless, students’ mathematical creativity has been explored using different perspectives in other studies (e.g., Leikin, 2013; Torrance, 1966; Dehaene, Spelke, Pinel, Stanescu & Tsivkin, 1999). Focusing on quantitative measures, researchers have been implementing three Torrance (1966) categories in their studies:

- Fluency (“the number of appropriate ways produced for solving a problem” (p.391)),
- Flexibility (“different groups of ways of solving” (p.391)), and
- Originality (“conventionality of a solution in a particular group of students with a similar educational history” (p.392))

Leikin (2009), for example, used a point system to evaluate these categories in students’ work. In this system, the “originality” measurement is given a high score if the solution produced is only prevalent among 15% or less of all solutions produced with a group of students. While Leikin acknowledged that solutions must be “appropriate” – “The notion of appropriateness has replaced the notion of correctness” (p.391), it seemed that an expert (e.g., an instructor or a researcher) was the one who made the judgment on what is or should be appropriate or original. With our perspective on mathematical creativity, we problematize such instances and aim to shift our focus to the producers of such solutions – the students. Our aim is to understand students’ perspectives on their own mathematical creativity. However, we notice that there is a need to first explore students’ perceptions of mathematical creativity, particularly at the tertiary level.

While there is research on mathematicians’ and mathematics instructors’ perceptions on mathematical creativity (Karakok et al., 2015; Borwein, Liljedahl & Zhai, 2014; Hadamard, 1945; Sriraman, 2009), research on students’ perceptions on mathematical creativity has received less attention. In one of our earlier studies, we examined students’ and
mathematicians’ definitions of mathematical creativity using three process categories: taking risks, making connections, and creating ideas (Tang et al., 2015). We found that students rarely associated making connections with mathematical creativity (9% of responses), whereas the mathematicians’ responses associated with making connections was about 38%. This study alerted us to think about explicitly valuing and discussing the processes that are deemed to be important in developing mathematical creativity in a classroom setting.

In this paper, we share how we approached this objective while exploring the following research questions:

- What are tertiary students’ perceptions of mathematical creativity?
- In what ways do these views evolve in an introduction-to-proofs course which emphasized mathematical creativity?

**METHODS**

To address the research question, we collected data in an introduction-to-proofs course at a small liberal arts college in southwestern United States. This course is typically taken by mathematics majors or minors in their second or third year and includes topics such as sets, logic and various proof techniques (e.g., direct proof, contradiction, contraposition and induction). The course was taught using an inquiry-based learning (IBL) pedagogical approach - students often worked in small groups and gave presentations to the class on their proofs.

The course explicitly valued creativity by making use of the Creativity-in-Progress Rubric (CPR) on Proving (Savic et al., 2107; Omar et al., 2019; Karakok et al., 2016), which is a formative assessment tool that students can use to persevere in proving and encourage creative processes. The rubric has two main categories: making connections and taking risks. The instructor gave assignments and exam questions where students had to use the rubric to assess their own or other’s work. Students also used reflection assignments to think about their definitions of mathematical creativity. Students used their responses to discuss their definitions with their groups.

At the end of the semester, 4 female and 3 male students agreed to be interviewed. Each student participated in a 60 – 90 minute semi-structured interview where they were asked to describe the course, discuss their views on creativity, and discuss the use of the CPR in the course. During the interview, students were also asked to compare their current views of mathematical creativity to the previous ones they shared on reflection assignments or pre-survey data and discuss, if possible, reasons for such changes. The interview protocol can be found in the appendix.

Interviews were coded using hypothesis coding (Saldaña, 2013) and five categories were extracted from the research questions of a larger project; one of the categories being creativity. Three of the seven participants’ transcripts were coded separately by the first and second author with 97% agreement. Because of this high degree of interrater reliability, the rest of the transcripts were coded by the first author only. A second-level coding was then done, where all utterances coded for creativity were coded with views, if a student was espousing their view of mathematical creativity, or evolution, if a student was speaking to how their views had evolved or changed after the course.
RESULTS

Students’ Views on Mathematical Creativity
Many of the students in this study expressed views on creativity that were strikingly similar to the three Torrance categories that we discussed in the previous section. For example, three of the students often talked about creativity as having a component of originality, as highlighted by the following quotes:

“What I think is, it’s mostly being able to, I guess bring your own… ideas to the table. Like, um, kind of doing something that no one else has done, or like figuring something out in a way someone else probably didn’t figure it out, or like working off another person’s ideas.” (Cargo)

Being creative in mathematics is the same as being creative in anything else. It’s taking the road less traveled. It’s not just doing what the herd is doing but finding your own way to get to where you need to be. (Stephanie)

“I would say to be creative in mathematics is basically anything, if you were ever faced with a problem you don’t really wanna stick with the generic, or you don’t want to find the generic way to answer it. You wanna find a way to solve the problem on your own by whatever means you can, as long as it works… and consistently works.” (Peyton)

Stephanie demonstrated how her search for something original when writing a mathematical proof turned into a moment of surprise and enjoyment:

“There were a lot of moments where you just almost stumble across something and you work through it and it ends up working and it’s completely different than what the other students had done. And it’s exciting.” (Stephanie)

We coded Olivia’s perspective to include flexibility and originality when asked about her views on creativity:

“My personal definition of creativity, and I guess to just really sum it up in one statement is just really thinking outside of the box and being able to be comfortable or at least willing to take risks. And, um, not just follow a standard format or a standard procedure, but being willing to be flexible and try different approaches, something you wouldn’t normally try, and, um, ya I guess that’s pretty much how I would describe it is just being able to be flexible and think abstract, think of something out of the ordinary.

Whereas, Stephanie’s perspective seems to relate to fluency as she spoke about solving a problem in multiple ways:

“So, in most lecture-based classes you’re taught this is how you do it. But as you get into the higher mathematics I’ve found that you can make connections from one to the other and you can solve things in different ways. Instead of using a calc trick to solve a problem, I might use a trig trick or just the geometric equations to solve something rather than doing a whole integral.

The students’ perspectives also included making connections as an important piece of mathematical creativity. For example, Olivia said:

“So, in that I think it [referring to IBL] forces you to really try to make connections and it forces you to get creative because you have, um, very little like understanding of the right way to do it, so it kind of throws that out of a student’s mind, out of my mind. And so it makes anything possible.

1 The names used in this paper are pseudonyms chosen by the students.
It is very possible that the student perspectives on creativity outlined above were highly influenced by the course and the use of the CPR on Proving. While there was no explicit mention of definitions of creativity in this course, the CPR on Proving was developed through a review of the literature as well as by asking mathematicians and students what mathematical creativity meant to them (Karakok et al., 2016). Therefore, these research-based ideas were present in the rubric either explicitly or implicitly. For example, one of the main categories of the rubric is called making connections and another subcategory was flexibility; so students may have been adopting the rubric language to describe their ideas on mathematical creativity at the time of the interview.

Students also had views on creativity that differed greatly from the definitions given in the literature. Several students spoke about the inherency of creativity, taking the perspective that it is an ability that you are born with and is fixed.

Um, to me creativity, that’s kind of like, born with—is like being able to come up with like a nifty idea for like a creative like art project that will make it like simple or like being able to- I know art takes like a lot of practice and a lot of work, like, itself to do—to be able to like draw or like paint. (Alice)

Vladmir also explained that recognising mistakes and evaluating them could lead to creativity:

I think that like when you mess up, you know when you mess up, the first thing you wanna do is you find out why, why or how, right? And you go back, and sometimes I think it’s when you go back and you’re forced to look the second or third time, that’s usually when you find like that separate path you know that might lead to like a creative path to get to your answer. (Vladmir)

Finally, three of the students saw creativity as akin to efficiency; that is, the shorter the proof was, the more they saw it as creative. In fact, two students spoke of another student in the class as being the most creative, since his proofs were the shortest and often made use of tricks that others had not thought of.

So, the one guy I was telling you about before, he was very efficient. He would make these algebraic tricks up, and then another person would come up with an algebraic trick to use. So, his creative moment, I could then use to expand on and do something a little different with to have my own creative moment. (Stephanie)

That’s interesting too. [laughs] That’s very, it seems so simple to come up with the n plus 1 squared is obviously less than n plus 1 squared times something else that’s positive. And by that, just by that simple first step they were able to come up with the proof. But it really only took that one little thing… Ya that’s very cool. That’s very short too, very efficient. (Peyton)

Evolution of Students’ Creativity Perspectives
Three of the students reported an explicit shift in how they thought about creativity or how they saw themselves as creative. These changing perspectives stemmed from different sources for different students. For Stephanie, a change in her view of mathematical creativity was due to having more tools to work with now that she had taken a class on proofs:

I think I started to look at creativity a little bit different through this course…Prior to this it’s been all very applied mathematics…So before, just using the trig equations to solve geometry was creative for me. Whereas now, this has just opened up a whole new door
of opportunities for it because I can solve a proof using a contradiction, while somebody else used a contrapositive and somebody else used a direct proof and somebody else used induction, and we all do it completely different.

For other students, shifts were attributed to the classroom community and the way that the course was structured. Olivia spoke of this when she mentioned:

We kind of all went in with kind of not really feeling confident in our abilities to be creative, so it was really interesting to see students that were quiet, reserved early on like show their work later in the semester and they had done something like totally cool and amazing... And seeing their involvement increase as the semester went on. So I feel you know their ability, like their confidence levels went up and I could say that’s true of me as well. So I wanna say that it’s, you know it wasn’t that like all the creative people took this course because I didn’t consider myself creative and I took the course, and I would say that that’s probably true of other students as well.

Stephanie echoed Olivia’s comment almost exactly:

At the beginning of the semester, I think a lot of people in that class were very shy and quiet, and so it was kind of hard to judge where their creativity was because they weren’t sharing it as much. Um, by the end of the course you had everybody speaking, you had everybody giving their opinions and how to work on things together, and you saw everyone grow. You saw everyone coming up with their own tools and tricks. And everyone was posing questions, not just the few of us that were outspoken to begin with. So you definitely saw growth in the class, um not only with the shyness but with the creativity, and coming up with their own ideas to change things and make them better.

Since the course was taught using IBL, students were encouraged to present their work to each other, especially if they approached a problem using a different method, thus some of these shifts seemed to be a result of seeing others’ as creative and reflecting it back on themselves. For instance, Peyton said:

I really, I really did not feel like I was being creative at all throughout the course. It really was just things in my head, it makes sense that led to a conclusion that made sense. But, considering that I thought other people were exceptionally creative, I kind of thought that maybe they though that about me too.

The most striking change is evinced by Peyton who went from not seeing mathematics as a creative subject to enjoying the creativity in mathematics.

Interviewer: And in your reflections you said something about, um, ‘I think I am on the spectrum that generally believes that, believes there is no need for creativity in mathematics. That’s been a key reason why I enjoy math. I know, I know if I get the answer then I have done it correct. There is a set process and if I learn the process then I’ll be able, I’ll’ – what’s that – ‘I’ll be successful’. So, do you wanna comment on that part?

Peyton: I… should have made that more in the past tense, because I believed that prior to taking this course. Um, but ya generally in past I figured, ‘cause math has always been lecture-based. There has been, you can figure out problems and it’s creative in the sense that you can figure out how, where you wanna start with the problem. But I like being able to know that if I am doing it correctly, the process correctly, then I will get to the answer. If I just repeat the process over and over then I know I’m going to learn it, which I do enjoy. I enjoy knowing when I’m gonna do something correctly as opposed to just spending a lot of time and then not even knowing if it’s gonna yield good results. But this course changed that quite a bit, because there really was no assurance that anything would be correct, but it still… required me to use different thought processes to get to a result hoping for the best, which was stressful to say the least, but still, it was fun.
DISCUSSION

In this study, we noticed that students’ view on mathematical creativity were centred around categories such as originality, flexibility and fluency as well as processes such as making connections, recognizing and evaluating mistakes, and mathematically observing other’s approaches. There were some students who still viewed creativity as an ability that a person has from birth, even though other students pointed out the possibility of development of creativity. Overall, students had a variety of views on creativity and, for many of the students, these views changed over the semester as they saw the instructor and other students focus on creativity and demonstrate creative mathematical practices. While Moore-Russo and Demler (2018) did examine students’ views of mathematical creativity at the tertiary level, their study was conducted with pre-service teachers. Our study is the first one to observe undergraduate students views of creativity and to determine how these views change throughout a semester.

These results also suggest that it is possible to affect students views on mathematical creativity through teaching practices. In our study (Karakok et al., 2015), we found that mathematicians believed creativity to be essential to their work yet didn’t teach it in their courses or feel that their students particularly saw mathematics as a creative discipline. This study, while small, has implications for teaching as, it seems, that this particular instructor’s course design aimed to explicitly value and foster students’ mathematical creativity and facilitated the evolvement of students’ perspective on mathematical creativity. We believe that this particular observation, namely, the connection between course design and teachers’ actions and ever-changing students’ perspectives on mathematical creativity requires additional exploration. In particular, which teacher actions are more fruitful to afford such changes and what other course design features contribute to these changes are important questions to explore at the tertiary level mathematics courses.

REFERENCES


APPENDIX

Interview Questions:

1. Class-Intro Question to get students talking, keep it short:
   - What other courses did you take this semester?
     a. Can you tell me more about how each course was taught?
     b. Did those courses influence your work in M305?
     c. Can you explain in what ways did or did not influence?
     d. Did this course influence your work in those classes? In what ways?

2. Definition of creativity to get students to think about creativity and trying to capture their “authentic” definition:
   - Can you tell us what it means to you to be creative in mathematics?
   - In your reflection you said, “I can expand on that?”
   - Did you feel creative in this course?
     o Can you tell me about it?
     o Why do you think you were creative?

   This question has two levels- their definition and if they feel they are creative.

3. Creativity in this course to elicit a) some teacher actions that they felt fostered creativity or b) advice on how to build a classroom to foster creativity
   - (Use their moment of creativity from previous question: which aspects of the course contributed this particular “creative moment” or other times you felt creative and how?)
   - In your opinion, which other aspect(s) could contribute to a student’s mathematical creativity?

4. Examine other students’ proofs trying to triangulate their definition in action with 3 students’ proof product:
   In this question I would like for you to read this theorem, please.
   a. I don’t want you to prove this question, but look at other students’ proofs and tell me what you think about their proofs. What do you think about this proof, which was constructed by another student? (keep it open ended like this first to get everything they can say-proofs will be emailed at the time of the interview. After they some stuff focus it to creativity)
   b. In particular, do you think it is creative? Why?
   c. How does this match your definition/perspective of creativity?

5. Rubric to see how students use the rubric, to tease out some product and process of creativity definitions/ideas from students’ perspective
   - How would you evaluate these students’ proofs using the rubric?

6. Rubric use in their proof process/course to have them focus on their use of the rubric if they haven’t already
   - How did you personally use the rubric while you were working on a proof?
     a. Which aspects/categories were useful to you and why?
     b. Which aspects/categories were challenging for you to use and why?
     c. Did you use the rubric or ideas from this rubric in another courses?
     d. If needed: How can we improve it for students’ use?
     e. If needed: How can we improve its use in classroom?

7. General Evaluation of Creativity in Math Courses
   - You know in courses instructors evaluate students’ learning in various ways, such as exams, in class participation, homework and such. Do you think students’ creativity should be or could be graded or evaluated?
     a. Why/why not?
     b. Do you think this rubric would help?