Communicative Artifacts of Proof: Transitions from Ascertaining to Persuading

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Abstract: With this poster, we wish to highlight an important aspect of the proving process. Specifically, we revisit Harel and Sowder's (1998, 2007) proof schemes to extend the authors' constructs of ascertaining and persuading. With this discussion, we reflect on the original theoretical framework in light of more recent research in the field and draw focus to a critical aspect of the proving process in which the prover generates the communicative artifacts of proof (CAP) critical to shifts between ascertaining to persuading. We also discuss possible ways in which an attention to the psychological and social activities involved in the development of the CAP might inform research and instruction.

Keywords: Communicating Proof, Ascertaining, Persuading

With their influential work, Harel and Sowder (2007) outlined a perspective for viewing proof in which the authors distinguish between two primary subprocesses in the proving process – ascertaining and persuading.

> Seldom do these processes occur in separation. Among mathematically experienced people and in a classroom environment conducive to intellectual interactions among the students and between the students and the teacher, when one ascertains for oneself, it is most likely that one would consider how to convince others, and vice versa. Thus, proving emerges as a response to cognitive-social needs, rather than exclusively to cognitive needs or social needs - a view consistent with Cobb and Yackel's *emergent perspective* (p. 6, 2007).

As stated, the authors situate these subprocesses of proof relative to a broader community. This perspective emphasized both the individual's reasoning to gain conviction about the validity or invalidity of a conjecture and the individual's communication of his or her thinking. Importantly, the authors point out that ascertaining and persuading often occur simultaneously, underscoring the anticipation of communicating a person's reasoning in a community.

This aspect of the "proof schemes" framework connects well with more recent research emphasizing socially situated aspects of the proving process. For instance, Stylianides' (2007) provided a way of defining proof relative to the classroom community with three properties: "set of accepted statements…modes of argumentation…modes of argument representation" (pp. 291-292). In his discussion, Stylianides pointed out that individuals within a community may not agree on valid reasoning or types of arguments. While investigating mathematicians' practices, Weber (2008) also emphasized the importance of the community in which an argument is presented when considering the argument's validity. As Harel and Sowder stated in the excerpt, these points are consistent with an emergent perspective, which holds at the fore the development of mathematical practices as individuals' participate in mathematical communities. More recently, Weber (2010) maintained the emergent perspective by discussing the explanatory power of proof, focusing on the importance of the audience's interpretation of a proof as the source of any proof's explanatory power.
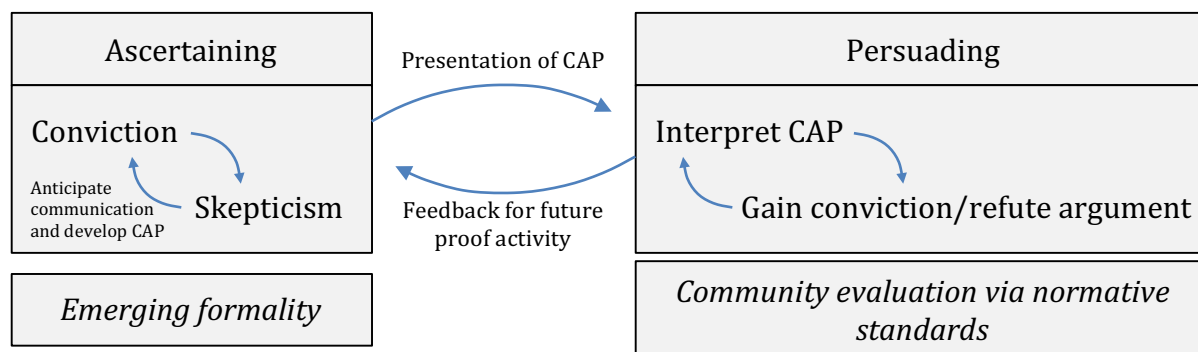
Figure 1: Diagram of interplay between individual and proof community

Our Theoretical Hypothesis on Ascertaining and Persuading

We envision the process of the development of a communicative artifact of proof as a result of the interplay between an individual and the proof community (Figure 1). While a version of this interplay may generalize to model any type of communication, we are focused on the dynamics involved in proof production. Specifically, we hypothesize that individuals' subprocess of ascertaining during proof production involves a cycle in which the individual balances conviction with skepticism. Throughout this process, the individual might anticipate the communication of their ideas within a broader community – anticipation that would likely inform the ascertaining subprocess, particularly during moments of skepticism, and persuading subprocess during the development and presentation of the communicative artifacts of proof (CAP).

The notion of CAP should inform the field's understanding and investigation of proof and the proving process by allowing proof researchers to distinguish between specific aspects of proof and focus on the specific proof activity in a participant's proving process. Further, these early notions of CAP can be developed to better explicate the types activity constituting the subprocesses of ascertaining and persuading. Refinement of the proving process may allow both researchers and instructors pinpoint hardships that students experience in their proving process, or may allow students to specifically target self-evaluation of their own proving.

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