Chapter 3
Formative Assessment of Creativity in Undergraduate Mathematics: Using a Creativity-in-Progress Rubric (CPR) on Proving

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Abstract Creativity is one of the most important aspects of mathematicians' work (Sriraman 2004), whether it is an enlightenment that is somewhat unexpected or a product that is aesthetically pleasing (Borwein, Liljedahl & Zhai 2014). There are studies in the primary and secondary levels on mathematical creativity of students (e.g., Leikin 2009; Silver 1997), and recent efforts have included mathematical creativity in K-12 education standards (e.g., Askew 2013). However, there is little research in undergraduate mathematics education on creativity. The project described in this chapter introduces an assessment framework for mathematical creativity in undergraduate mathematics teaching and learning. One outcome of this project is a formative assessment tool, the Creativity-in-Progress Rubric (CPR) on proving, that can be implemented in an introductory proof course. Using multiple methodological tools on a case study, we demonstrate how implementing the CPR on proving can help researchers and educators to observe and assess a student's development of mathematical creativity in proving. We claim if mathematicians who regularly engage in proving value creativity, then there should be some explicit discussion of mathematical creativity in proving early in a young mathematician's career. In this chapter, we also outline suggestions on how to introduce mathematical creativity in the undergraduate classroom.

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R. Leikin, B. Sriraman (eds.), *Creativity and Giftedness*, Advances in Mathematics Education, DOI 10.1007/978-3-319-38840-3_3

Keywords Formative assessment • Mathematical creativity • Proof • Undergraduate mathematics education

3.1 Introduction

Though creativity is an important aspect of professional mathematicians' work (Borwein et al. 2014), it is a complicated subject for mathematics educators to research, given that there are over 100 different definitions of creativity (Mann 2006). A considerable amount of literature concentrates on mathematical creativity at the primary and secondary levels (e.g., Silver 1997; Lev-Zamir and Leikin 2011). However, our examination of research at the tertiary, or post-secondary, level revealed little discussion of how students are creative or how creativity can be fostered in undergraduate courses, particularly in proving or proof-based courses. Given that the students in tertiary courses are the next generation of mathematicians, engineers, or math educators, developing their mathematical creativity is crucial. Mann (2006) stated that avoiding the acknowledgment of creativity could "drive the creatively talented underground or, worse yet, cause them to give up the study of mathematics altogether" (p. 239). Although Mann referenced secondary students, this recognition is important for young mathematicians at all levels, including post-secondary students.

Aiming to spark discussions about creativity at the tertiary levels, our focus in valuing creativity during the proving process yielded a Creativity-in-Progress Rubric on Proving (see Table 3.1). The rubric is intended to be used as a formative assessment, so that students can improve their metacognition in proving, and eventually, their final proofs. We begin this discussion by introducing the pertinent literature and theoretical framework involved. The rubric is introduced with explanations of its two main categories: *Making Connections* and *Taking Risks*. Two case studies are presented to further illustrate these categories. Finally, we discuss observations of the case studies, implications of utilizing the rubric as a formative assessment, suggestions of additional ways of implementing it in a proof-based tertiary course, and provide future research with the rubric.

3.2 Background

3.2.1 Perspectives of Creativity

Researching an individual's creativity and the ways in which to enhance it has been an endeavor yielding many definitions and approaches. For example, Kozbelt et al. (2010) provided a summary of contemporary theories through a meta-analysis and outlined ten major perspectives of creativity: Developmental, Psychometric, Economics, Stage and Componential Process, Cognitive, Problem Solving and

Table 3.1 Creativity-in-Progress Rubric (CPR) on proving

MAKING CONNECTIONS:	Beginning	Developing	Advancing
Between Definitions/Theorems	Recognizes some relevant definitions/theorems from the course or textbook with no attempts to connect them in their proving	Recognizes some relevant definitions/theorems from the course and attempts to connect them in their proving	Implements relevant definitions/theorems from the course and/or other resources outside the course in their proving
Between Representations[1]	Provides a representation with no attempts to connect it to another representation	Provides multiple representations and recognizes connections between representations	Provides multiple representations and uses connections between different representations
Between Examples	Generates one or two specific examples with no attempt to connect them	Generates one or two specific examples and recognizes a connection between them	Generates several specific examples and uses the key idea synthesized from their generation
TAKING RISKS:	**Beginning**	**Developing**	**Advancing**
Tools and Tricks	Uses a tool or trick that is algorithmic or conventional for the course or the student	Uses a tool or trick that is model-based or partly unconventional for the course or the student	Creates a tool or trick that is unconventional for the course or the student
Flexibility	Attempts one proof technique	Acknowledges the possibility of different proving approaches, but attempts no further examination	Acts on different proving approaches
Perseverance	Begins to engage with proving	Continues to engage with surface level features but not with the key ideas	Continues to engage with the key ideas
Posing Questions	Recognizes a question should be asked, but does not formulate a question	Poses questions clarifying a statement of a definition or theorem	Poses questions about reasoning within a proof
Evaluation of the Proof Attempt	Checks work locally	Recognizes a successful or unsuccessful proving attempt	Recognizes the key idea that makes the proving attempt successful or unsuccessful

[1] We define a *mathematical representation* similar to NCTM's (2000) definition. It includes written work in the form of diagrams, graphical displays, and symbolic expressions. We also include linguistic expressions as a form of lexical or oral representation. For example, a student can use the lexical or oral representation, "the intersection of sets A and B"; a Venn Diagram to depict his/her mathematical thinking; a symbolic representation $A \cap B$; or set notation $\{x | x \in A \text{ and } x \in B\}$ (which is also a symbolic representation). Note the last two representations are in the same category, e.g. symbolic, but they are still considered two different representations.

Expertise-Based, Problem Finding, Evolutionary, Typological, and Systems. All of these theoretical approaches focus mainly on *domain-general* creativity, or "the creative ability to generate divergent or original ideas in a wide variety of domains" (Hong and Milgram 2010, p. 272). However, Baer (1998) cautioned against studying only general creativity practices; instead he advocated for research in *domain-specific* creativity. Plucker and Zabelina (2009) built on this idea: "some even argue that creativity is not only domain-specific, but that it is necessary to define specific ability differences within domains and even on specific tasks" (p. 6).

In both domain-specific and domain-general studies, creativity researchers either focus on the *end product* as original and useful (Runco and Jaeger 2012), or on a *process* that involves convergent or divergent thinking (Guilford 1967). Even though researching the creative process creates difficult hypotheses for testing (Torrance 1966), researching the creative product may not provide full understanding of the development of creativity, or may not reflect the creativity used to reach that product. For example, Pelczer and Rodriguez (2011) pointed out that "it is important that when judging the creativity of a student we pay attention also to the process by which [s/]he arrived to the results and not only to the final problem" (p. 394).

Viewing personal creativity as a product or a process brings up the following question: *For whom* is the product or process creative? This issue has been acknowledged through the discussion of *relative* and *absolute* creativity. Relative creativity is described as “the discoveries by a specific person within a specific reference group, to human imagination that creates something new” (Vygotsky 1982, 1984; as cited by Leikin 2009, p. 131). That is, a person may create something that is new to him/her or to his/her peers in a given subculture, but it may not be new to the community of more knowledgeable others. Absolute creativity, on the other hand, considers discoveries at a global level, such as the proof of Fermat’s Last Theorem by Andrew Wiles (1995). In the education field, Liljedahl (2013) stated, “students have moments of creativity that may, or may not, result in the creation of a product that may, or may not, be either useful or novel” (p. 256). Thus, it is reasonable that the “relative creativity” perspective has been implemented frequently in previous educational creativity research (e.g., Liljedahl and Sriraman 2006; Leikin 2009).

3.2.2 What Is Mathematical Creativity?

Early researchers aimed to define mathematical creativity by focusing on experts. For example, Hadamard (1945) explored mathematical creativity of prominent mathematicians across the world through the use of surveys by mail. He theorized that the four stages the psychologist Wallas (1926) conjectured were applicable in describing the work of a mathematician. The four stages are preparation (thoroughly understanding the problem), incubation (when the mind solves a problem subconsciously and automatically), illumination (internally generating an idea after the incubation process), and verification (determining if that idea is correct).

However, Guilford (1950) found Hadamard’s stages “superficial from the psychological point of view” (p. 451). He was concerned that these stages were not providing sufficient detail about the mental processes that occur. Guilford, then, created a list of testable factors that were later refined by other researchers: *fluency*, *flexibility*, *originality*, and *elaboration. Fluency* refers to the “number of ideas generated in response to a prompt” (Silver 1997, p. 76). *Flexibility* is the ability to shift approaches when the current approach is unproductive for generating a response to a prompt (Silver 1997). *Originality* (or novelty) is described as the ability to create a unique production or an unusual thought (Torrance 1966). *Elaboration* refers to the ability to produce a detailed plan and generalize ideas (Torrance, ibid). These factors of creativity have been used at the primary and secondary stages of schooling to determine students’ levels of creativity (e.g., Balka 1974; Leikin 2009).

3.2.3 Mathematical Creativity at the Tertiary Level

There are on-going efforts to introduce challenging tasks in tertiary mathematics [AU1] courses (e.g., through implementation of new pedagogical strategies such as inquiry-based learning (Smith 2006) or realistic mathematics education (Gravemeijer and Doorman 1999). Such tasks are useful to elicit students’ mathematical creativity

(Leikin 2014). Zazkis and Holton (2009) suggested mathematical problems that could challenge students at this level to possibly promote creative processes, and provided an outline of the importance of mathematical creativity. However, we know little about how to explicitly value tertiary students' creativity when such mathematical problems are implemented.

An essential aspect in tertiary mathematics is to ask students to communicate their reasoning through written proofs. Alcock and Weber (2008) stated that "a central unresolved issue in mathematics education is that of how to help students develop their conceptions of proof and ability to write proofs" (p. 101). While there is research about proving from many different perspectives (e.g., Selden and Selden 1995; Harel and Sowder 1998; Weber 2001), few have investigated mathematical creativity in proving (e.g., Leikin 2014).

To address this need, our research group developed a formative assessment tool that could be used to promote each student's development of mathematical creativity on a given mathematical task in a proof-based course and to examine this development over the duration of the course. The Creativity-in-Progress Rubric (*CPR*) on Proving can inform both students and teachers about the progression that a student is making in developing his/her own mathematical creativity.

3.3 Creativity-in-Progress Rubric on Proving

3.3.1 Development of CPR on Proving

Development of the CPR on Proving was motivated by the aforementioned studies, as well as our investigation of mathematicians' perspectives on students' mathematical creativity in tertiary level courses (Karakok et al. 2016). We interviewed six active research mathematicians (with pseudonyms Drs. A-F), who teach undergraduate and graduate level mathematics courses, and asked them about the role of mathematical creativity in proving, in teaching mathematics, and in students' learning. The mathematicians in our study also examined three student-created proofs of a theorem in number theory (Birky et al. 2011) using a domain-general creative thinking rubric (Rhodes 2010) created by the American Association of Colleges and Universities (AAC&U). This domain-general rubric was created to record growth and value creativity in a broad range of interdisciplinary student work samples. We utilized the mathematicians' ideas to modify the AAC&U rubric to make it domain-specific to mathematics. Our modification was also influenced by Leikin's (2009) rubric on mathematical creativity in problem solving, since the proving process is considered a subset of the problem-solving process (Furinghetti and Morselli 2009). We leveraged these ideas to develop a rubric (Savic et al. 2015) which we then refined using students' interview data (Tang et al. 2015).

Our development and refinement of the CPR on Proving were grounded in two of the ten aforementioned perspectives of creativity: *Developmental* and *Problem Solving and Expertise-Based* (Kozbelt et al. 2010). The primary assertion of the creativity theories in the *Developmental* perspective is that creativity develops over time, and the main focus of investigation is a person's developing process of creativity. This perspective also emphasizes the role of the environment surrounding a student, in which interactive elements occur to enhance a student's creativity. The second perspective that helped shape our project is *Problem Solving and Expertise-Based*, which emphasizes the role of an individual's problem-solving process and also argues that "creative thought ultimately stems from mundane cognitive processes" (Kozbelt et al. 2010, p. 33). This particular idea highlights that during problem solving or proving, implementing seemingly "mundane" tasks (such as finding relevant examples or representing the same concept in multiple ways) help the development of creativity by laying the foundations for creativity in novel situations. For example, Kozbelt et al. (2010) noted that "archival study of individual creative episodes of eminent scientists has generated a number of computational models of the creative process" (p. 33). These computational models included key components such as problem-solving processes, heuristics (ways that experts solve problems), and tasks. Furthermore, this perspective underscores the use of open-ended problems to challenge students' thinking processes, providing opportunities for students to use experts' ways of solving problems to be creative in such novel situations.

Overall, the CPR on Proving was developed from a relative, domain-specific approach, focusing on an individual student's progress on tasks and the development of his/her creativity over time. The next section provides a brief description of each of the categories developed.

3.3.2 Categories and Levels of the CPR on Proving

The CPR on Proving has two categories: *Making Connections* and *Taking Risks,* which are divided into subcategories that are reflective of the different aspects of creativity found in prior research. For each subcategory, the rubric provides three general levels: *Beginning*, *Developing*, and *Advancing*, each of which serves as a marker along the continuum of a student's progress in that subcategory. This continuum among levels of the rubric communicates the possible states of growth. Our research group acknowledges that some students may exhibit qualities that place them further along in one level, or in between two levels. Hence, the continuum of levels for each subcategory allows for a better approximation of placing proving attempts on the rubric based on the work provided. The user of the rubric can indicate the corresponding level by tracing the arrow using a highlighter or a marker (for example, see Fig. 3.2 in Sect. 3.4).

The descriptions of the subcategories in Table 3.1 are derived from either the research literature on creativity, quotes from our study of creativity with mathematicians (Karakok et al. 2015), or both. The description of the rubric is followed by two case studies in Sect. 3.4, which will provide further examples for each subcategory.

3.3.2.1 Making Connections

During the proving process, a student should be encouraged to make connections from previously learned material and apply these connections to new tasks. This category originated from the AAC&U Creative Thinking Rubric (Rhodes 2010) category *Connecting, Synthesizing, Transforming*, where a milestone level is achieved by a student who "connects ideas or solutions into novel ways" (p. 2). In our prior studies (Karakok et al. 2015; Tang et al. 2015), mathematicians commented that connecting ideas from other areas of mathematics was a crucial process in their work. For example, Dr. C. said,

> [F]inally I found some nice books in an area totally unrelated to mine, in matrix theory, and at some point I realized that I could apply this [aspect of Matrix Theory] that no one ever thought of applying to differential equations before and solved my problem … [I]n the process of applying it, I think I created … some new connections.

We define the category, **Making Connections**, as *the ability to connect the proving task with definitions, theorems, multiple representations, and examples from the current course that a student is in, and possible prior experiences from previous courses*. In this category, we consider making connections: between definitions/theorems, between representations, and between examples. Each of these subcategories is described below.

Between Definitions/Theorems To enhance connection-making abilities, students should make use of definitions/theorems previously discussed in the course and perhaps, from other courses. Dr. A, one of the mathematicians in our previous study (Karakok et al. 2015) stated, "Somehow your mind has to spread out a little bit to see…connections to other theorems you could use…That's creativity also." The way in which students use previous definitions/theorems in their proving processes defines the Beginning, Developing, or Advancing levels.

At the *beginning* level, a student recognizes some relevant (or irrelevant) definitions/theorems from the course or textbook (or resources related to the course) with no evidence of explicit attempts to connect those definitions/theorems to the task during the current proving process. For example, a student might list definitions related to a concept (e.g., function, onto, 1–1) that s/he has read in the task without providing any evidence of connecting these definitions to the proof in his/her attempts. At the *developing* level, a student recognizes some relevant definitions/theorems from the course or textbooks (or resources related to the course) with evidence of an attempt to connect these to the task during the proving process. At the *advancing* level, a student implements definitions/theorems from the course and/or other resources outside the course in his/her proving. While discussing creativity in the post-secondary classroom, Dr. E stated, "I think when students realize that they can solve these problems with things that are not just in this section. It can be from some other part of the course. Be somewhat creative." So, at the advancing level, a student not only recognizes relevant definitions and theorems, but also explicitly illustrates using them.

Between Representations Creating or using multiple representations can be important for solving or understanding problems. The National Council of Teachers of Mathematics (2000) referred to a representation as one way a student might depict his/her mathematical thinking. Other researchers have emphasized making connections among and between representations of a concept, such as representing a function as a table, graph, verbally and symbolically (Carlson et al. 2010). The connections students make between representations is also important for their development of mathematical creativity. For instance, Dr. F said, "[Creativity] is primarily to look at things differently. For example, notice that some equations result in some geometry, with that some geometry connects to some algebra." In this subcategory, students may not always make broad connections across the two fields of geometry and algebra, but perhaps attempt to utilize representations within each field.

Representations include written work in the form of diagrams, graphical displays, and symbolic expressions. We also include linguistic expressions, either oral or written, as a form of a lexical representation. For example, a student can write, "the intersection of sets A and B," or orally state "A intersect B is the set of common elements between A and B." A Venn Diagram, the symbolic representation $A \cap B$, the set notation $\{x | x \in A \text{ and } x \in B\}$ (which is also a symbolic representation), are other possible representations a student can use to depict his/her mathematical thinking about the concept of intersection. Note the last two representations are in the same category (i.e., symbolic), but they are still considered to be two different representations.

At the *beginning* level, a student is able to provide a representation with no evidence of attempting to connect it to another representation. At the *developing* level, a student should recognize connections between some representations and attempt to connect them to the proving task on hand. Students on this level may not recognize all the related representations of a mathematical object, but at least demonstrate connections to more than one representation. At the *advancing* level, a student should utilize and implement different representations in his/her proving process, hence making explicit connections between the different possible representations of a mathematical concept and applying these connections to their proof attempt.

Between Examples This subcategory refers to students' scratch work or "play time" where they experiment with different ideas to attempt the task. They can do this by creating examples, comparing and contrasting examples, or by providing counterexamples that are sufficient to disprove a claim. Students usually practice with examples as a method to understand the definition of a concept or to validate the verity of a mathematical statement. Doing so could help students to develop their creativity. Dr. A, for example, stated:

> Thinking to yourself: 'Can I make up a little example that'll help me get a sense of this?' 'Is there something I can try to do that will help me get my hands on this new concept?' And, again, I think that is a creative process.

However, students need to further their example generation to see possible connections to a pattern, which is somewhat of a difficulty for students (Dahlberg and Housman 1997). Merely asking students to create examples does not necessarily lead to a proof production. As Iannone et al. (2011) state, "[I]f example generation is to be a useful pedagogical strategy, more nuance is needed in its implementation" (p. 11). Thus, in this subcategory we aim to push students to make connections between examples to generalize to a key idea, or pattern.

At the *beginning* level, a student generates one or two specific examples with no attempt to connect them. However, at the *developing* level, a student recognizes a connection between the generated examples. At the *advancing* level, a student utilizes the key idea synthesized from generating examples. One way to see this is when students recognize patterns from examples and symbolize these patterns formally to assist in the proving process.

3.3.2.2 Taking Risks

During the proving process, a student should be encouraged to explore concepts, create new ideas, and evaluate those attempts in order to ultimately create a valid proof. Those explorations require a student to take risks during the proving process. The category Taking Risks originated from the AAC&U Creative Thinking Rubric (Rhodes 2010), where the highest level is achieved by a student who "[a]ctively seeks out and follows through on untested and potentially risky directions or approaches to the assignment in the final product" (p. 2). The category and the forthcoming subcategories were also influenced by our interviews with mathematicians about the proving process (Karakok et al. 2015; Tang et al. 2015). For example, Dr. B, stated:

> [O]ccasionally when you are trying to prove something, you know where you want to go, so it's just a matter of trying several different things, and seeing what fits in order to get you there. But other times, you don't know where you are going. Proving means you're saying, "There is this problem, and I'm going to try this approach and this approach. I don't even know what the next step should be." So I think the creativity part of it affects the proof differently.

Therefore, we define the category **Taking Risks** as the *ability to actively attempt a proof, perhaps using multiple proof approaches and/or techniques, posing questions about reasoning within the attempts, and evaluating those attempts*. The five subcategories, *Tools and Tricks, Flexibility, Perseverance, Posing Questions,* and *Evaluation of the Proof Attempt,* are described below.

Tools and Tricks We found through interviewing mathematicians that creativity also can involve creating tools or tricks in the proving process. Dr. E stated, "You can be very creative about the way in which you approach the question, either with new tools or with a really good idea for a partial result." Using these tools or tricks can be original to the student or the course, thus leading to *relative* creativity in their proving. A common example of a tool or trick that is original is involved in the proof of the theorem, “There are infinitely many prime numbers.” One must assume a finite amount of prime numbers, $p_1, \ldots, p_n$, and create a new number $(p_1 \cdot \ldots \cdot p_n + 1)$ that is larger than the largest prime p_n which one then shows is still prime. The usual question asked by students when presented with this proof is, “Where did this come from?” This new number is an example of an unexpected object (tool) created to assist with creation of the proof.

The creation of an entirely new tool or trick is creative, and we believe it is evidence of a risk taken; however, the tool or trick need not be original to be considered in line with creative thought. Adapting a previous tool or trick to new contexts is also considered unconventional. At the *beginning* level, a student uses a tool or trick that is algorithmic or conventional. Conventional solutions are “generally recommended by the curriculum, displayed in textbooks, and usually taught by the teachers” (Leikin 2009, p. 133). For example, if an instructor presented the trick that you should “add zero” while completing a square, a student at the beginning level would employ the same trick in a proof that required completing the square. At the *developing* level, a student uses a tool or trick that is model-based or partly unconventional. If the student used that trick in a new context or in a proof that did not require completing the square in the same course, the student would be considered developing. For example if the student had to prove $4 \mid \left(5^n - 1\right)$ for every natural number n, and in the inductive case wrote $5^{k+1} - 1 = \left(5^k - 1 + 1\right)5 - 1$, then the student would be considered at the developing level. Finally, at the *advancing* level, a student creates a tool or trick that is unconventional for the course or the student. If a student thought of “adding zero” without any prompting or previous knowledge in the course, this would be considered advancing.

Flexibility In the category Making Connections, we discussed recognizing the need to use a proof technique used on previous proofs on a new proof. Flexibility is the ability to shift approaches in proving a theorem or claim. This idea was adapted from Silver’s (1997) definition of flexibility for problem solving. For example, a student might begin a proof using a direct proof, but then shift to a proof by contradiction if the student did not find the first technique helpful. Dr. D found this ability helpful during her proof attempts, “If it doesn’t work you say ‘let me try something different and use some information I gathered to [come] up [with] something that might be more useful.’”

In this subcategory continuum, at the *beginning* level, a student attempts one proof technique in his/her proof. At the *developing* level, a student acknowledges the possibility of using different proof techniques, but does not act on it. Finally, at the *advancing* level, a student acts on different proving approaches. A student at the advancing level would act on multiple proof techniques, perhaps because the student did not find the initial proof technique(s) helpful, or s/he wanted to attempt a more efficient proof.

Perseverance Perseverance is a quality that many mathematicians possess either consciously or sub-consciously, and that many instructors want their students to exhibit in their courses. In his seminal work on problem solving, Schoenfeld (1992) pointed out that students "give up working on a problem after a few minutes of unsuccessful attempts even though they might have solved it had they persevered" (p. 359). Dr. B echoes the importance of perseverance in his statement:

> [T]he creativity part is, 'ok I know I'm going to get this far, and I want to get here, kind of four steps down, but somehow able to pinpoint the key idea,' so that's part of the creative process… sometimes it'll just be a matter of trying various [ideas] to get that [next] step.

We identify perseverance in a proving process of a student when he/she is continuing to engage in the proving process no matter the hardship. Time is not an aspect of our definition of perseverance; rather the engagement with key aspects and the challenges of the proof is the gauge of perseverance. According to Thom and Pirie (2002), perseverance is a "sense (i.e. intuitive and experiential) in knowing when to continue with, and not to give up too soon on a chosen strategy or action" (p. 2). Therefore, to develop creativity in proving, perseverance is needed.

At the *beginning* level, a student demonstrates perseverance by engaging with the proving process minimally. For example, a student would not finish his/her first proving attempt, and would not try any other proof attempt. At the *developing* level, a student would continue to engage with surface level features of the proving process, but there is no evidence of engagement with the key ideas of the proof. Finally, at the *advancing* level, a student perseveres by engaging with key ideas of the proof. S/he may or may not have a final valid proof, but is engaging with the key ideas or reasoning for the proof.

Posing Questions In the proving process, there are certain times when a question can lead to a creative thought. Dr. B acknowledged that, while researching, he asks himself, "What do I need to do in order to make that step so the rest of it is downhill?" Pelczer and Rodriguez (2011), citing Jensen (1973), stated that if students want to be creative, they "should be able to pose mathematical questions that allow exploration of the original problem" (p. 384). Posing questions can occur throughout the proving process, but there are different qualities of questions that students can pose.

At the *beginning* level, a student will recognize that a question should be asked (perhaps with a question mark next to his/her proof), but will not formulate a full question. At the *developing* level, a student will pose a clarifying question about a statement of a definition or theorem, for example to clarify terms used within a theorem statement. Finally, at the *advancing* level, a student will pose a clarifying question about the reasoning in the proof.

Evaluation of the Proof Attempt We define a successful proof as a correct proof which "establishes the truth of a theorem" (Selden and Selden 2003, p. 5). A *successful* proof is neither a necessary nor sufficient condition for a *creative* proof attempt. That being said, understanding the *key ideas* that make a proof attempt successful or unsuccessful can provide insight for future proof attempts. Key ideas are defined by Raman (2003) as, "a heuristic idea which one can map to a formal proof with appropriate sense of rigor. It … gives a sense of *understanding* and *conviction*. Key ideas show *why* a particular claim is true" (p. 323). For example, Dr. D stated that she re-evaluated a result to find a visual application and ended up "go[ing] back to the drawing board because the stuff that we thought we proved was wrong. My thinking about [a] different way to visualize it and seeing something completely unexpected got us there."

At the *beginning* level, a student is one who only checks work locally, that is, for small errors or typos. At the *developing* level, a student recognizes a successful or unsuccessful proof attempt without identifying the key idea that makes the attempt successful or unsuccessful. A student may look at his/her proof, realize that it is incorrect, but not realize exactly why the proof is incorrect. Finally, an ability to recognize the key idea in a proof attempt, successful or unsuccessful, describes an *advancing*-level in this subcategory.

In the next section, we provide two students' proving attempts on two different tasks to illustrate student work that falls into various levels of these subcategories.

3.4 Case Studies

The data presented in this section was collected in Spring 2014 at a large research university in the United States. In an inquiry-based, introduction-to-proof course, 24 students were given LiveScribe pens, a data collection tool capable of capturing audio and written work in real time (For details on LiveScribe pens, see Savic 2015). Use of this technology was an intentional attempt to capture the processes of students' proof development, including scratch work and verbal expressions. All students were required to do and turn in their homework using the pen and special paper; all homework was downloaded to the professor's computer for both grading and analysis. The LiveScribe pen data was examined for the eight students that participated in an exit interview to relate exit interview data with performance throughout the semester. Our research group narrowed the data analysis to five proving tasks that we jointly agreed could elicit creativity in the proving process. Those tasks were coded using the CPR on Proving. Here we report on two of those five tasks.

Theorem 29 is stated: "If 3 divides the sum of the digits of n, then 3 divides n". This theorem was the third theorem in the number theory section of the course, located after the definition of even and odd numbers, divisibility, (Definition S: $a \mid b \Leftrightarrow b = na \; for \; some \; n \in \mathbb{Z}$) and the following theorems (27 and 28): "If m and n are even numbers, prove that $m+n$ and $m \cdot n$ are even numbers" and "If $a \mid b$ and $a \mid c$, then $a \mid (br+cs)$ for any $r,s \in \mathbb{Z}$." respectively. We will refer to Theorem 29 as the "digit" theorem. The other task is the second theorem on the second test: "If 3 divides a natural number n, then n is a trapezoidal number." A trapezoidal number, defined on the test, is a natural number that can be expressed as the sum of two or more consecutive natural numbers. We will refer to Theorem 2 of Test 2 as the "trapezoid" theorem.

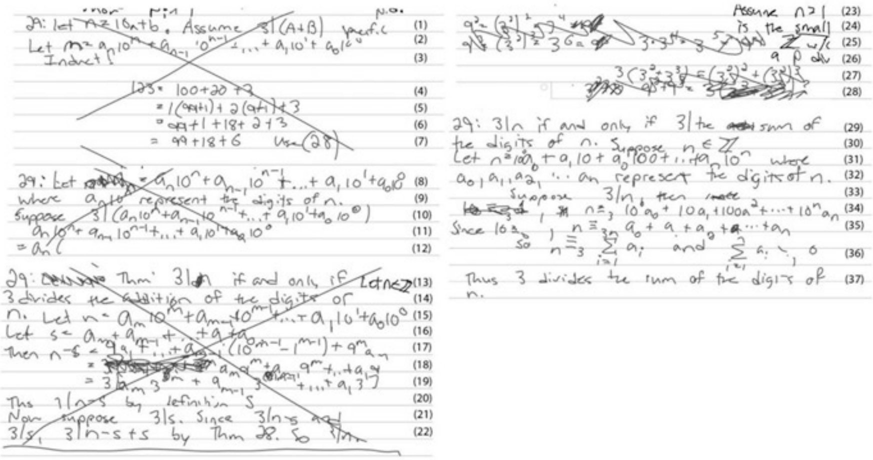

Fig. 3.1 Greg's proving process for the digit theorem

To demonstrate the aspects of mathematical creativity in proving that the CPR reveals, we focus on two students in the course, Greg and Marty. At the end of Spring of 2014, Greg was a mathematics major with enough credits to be a fourth-year student; Marty was a fourth year dual-major in Economics and History. When coding students' work for one proof task, we used a holistic approach to focus on the students' entire collection of attempts rather than each individual attempt. For instance, we coded Greg's four attempts of the digit theorem (see Fig. 3.1) rather than each individual attempt. We present both students' proving attempts for both theorems.

3.4.1 The Digit Theorem

All of the proving attempts by Greg for the digit theorem are located in Fig. 3.1. The LiveScribe Pen time-stamped each recording so we were able to see that the student attempted this proof at least four times (Fig. 3.1 Lines 1–7, Lines 8–12, Lines 13–28, and Lines 29–37) over two days. Figure 3.2 provides the aggregation of our coding of all of these attempts. We provide an explanation for the coding of each subcategory (written in italics) below.

Between Definitions/Theorems We observed that Greg used Definition S (Line 20) and Theorem 28 (Lines 7 and 22) and tried to implement both into his proving attempts. Therefore, since he implemented definitions and theorems in his proving attempts, we coded his work as "advancing". This is indicated with the dark arrow in Fig. 3.2.

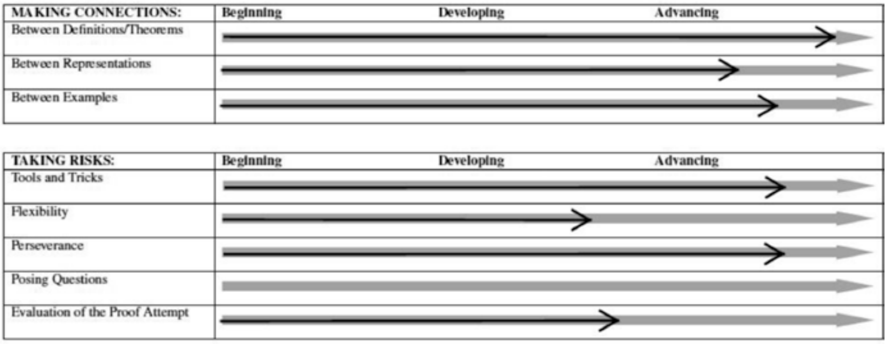

Fig. 3.2 Levels of Greg's work on the digit theorem

Between Representations Greg also had two symbolic representations for the natural number $n: a_n 10^n + a_{n-1} 10^{n-1} + \ldots + a_1 10^1 + a_0 10^0 \equiv_3 \sum_{i=1}^{n} a_i$ (Lines 34 and 36). We observed that he used connections between these two representations in his proving process, but it is unclear how the second representation was helpful to his attempt. Thus, his work is categorized in the "advancing" level, yet not as complete as in *Between Definitions/Theorems*.

Between Examples Greg explored an example, $n = 123$ (Lines 4–7), and when he tried to factor a 3 from 9^n (Line 19), he generated examples (Lines 23–28). Since he used the key idea generated from "123" (Lines 17–19), he demonstrated an "advancing" level in our coding.

Tools and Tricks Greg used a notation "$\equiv_3$", which means "equivalent modulo 3," that was unconventional for the course because it had not been previously discussed. He also used the trick of rewriting 10 as $9+1$ and 100 as $99+1$ (Lines 4 and 5). Therefore, we coded Greg's work as "advancing."

Flexibility On Line 3, he indicated that the proof might be approached using induction, but largely used direct proof, hence he acknowledged the possibility of a different approach with no further examination. For this reason we coded his work in the "developing" level.

Perseverance Greg engaged with key ideas of the proof (Lines 17–22), and also created many proving attempts, so he was coded as "advancing" in this subcategory.

Posing Questions There were no questions posed either in his written or oral recorded work. Due to this lack of evidence, no levels were assigned to this subcategory of his work.

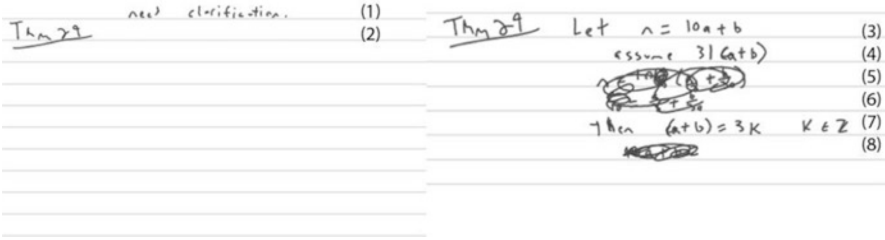

Fig. 3.3 Marty's proving process for the digit theorem

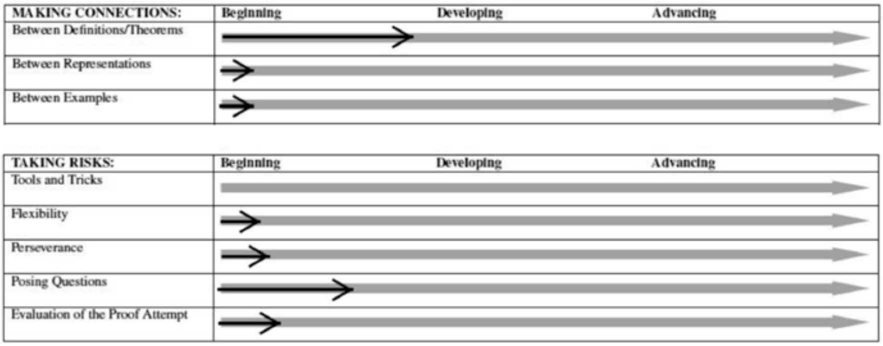

Fig. 3.4 Levels of Marty's work on the digit theorem

Evaluation of the Proof Attempt Though he recognized unsuccessful attempts by crossing them out, there is no explicit acknowledgment of the key idea(s) that made the attempt unsuccessful, thus his work was coded as "developing." However, the multiple evaluations of his proving contributed to the arrow being closer to advancing.

For the digit theorem, all of Marty's attempts are demonstrated in Fig. 3.3. Marty attempted the proof first (Fig. 3.3 Lines 1–2), and attempted the proof again two days later (Fig. 3.3 Lines 3–8). In Fig. 3.4, we present the summary of our coding of Marty on the digit theorem.

As it can be observed in Fig. 3.3, Marty's demonstrated proof process was brief. Thus, we share descriptions for some of the subcategories. Marty recognized but did not implement the definition of divisibility by stating that $(a+b)=3k$ (Line 7). Also, he wrote "need clarification," (Line 1) which was recognition to pose a question without fully formulating the question. Those two actions provided evidence for the positioning of the arrows in between the "beginning" and "developing" levels for the subcategories of *Between Definitions/Theorems* and *Posing Questions*. For other subcategories, we only observed "beginning" level actions, with the exception of the *Tools and Tricks* subcategory. Marty's work did not provide evidence for this subcategory, so no level is indicated.

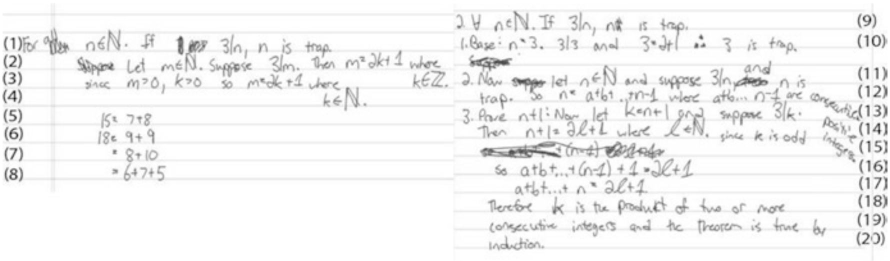

Fig. 3.5 Proving attempts of Greg on the trapezoid theorem

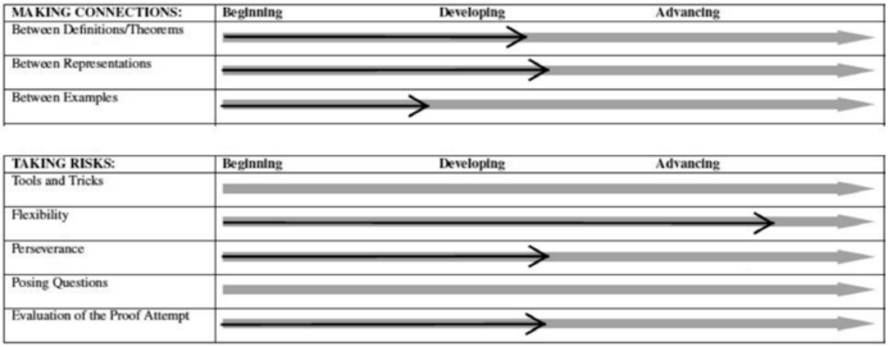

Fig. 3.6 Levels of Greg's work on the trapezoid theorem

3.4.2 The Trapezoid Theorem

The proving attempts of the trapezoid theorem from Greg are presented in Fig. 3.5, while the levels coded for Greg are in Fig. 3.6.

In the Making Connections category, Greg implemented definitions for trapezoidal (Line 12) and odd numbers (Line 14) in his proof, and attempted to use both definitions in his proving ("developing" in *Between Definitions/Theorems*). He used a representation of a trapezoidal number (Line 12) and attempted to connect it to the definition of a trapezoidal number ("developing" in *Between Representations*). Finally, he generated some examples (Lines 5–8), but there was no indication that he recognized a connection between his examples so as to generate a key idea to use in the proof attempt ("beginning" in *Between Examples*).

In the Taking Risks category, Greg did not use a trick or tool or pose a question (both left blank in the rubric). He acted on two different proving approaches, namely direct proof and induction ("advancing" in *Flexibility*). Greg continued to engage with the surface level, but not the key ideas, of the proof ("developing" in *Perseverance*). Finally, he evaluated his first proof attempt but did not recognize the key idea that made it unsuccessful ("developing" in *Evaluation of the Proof Attempt*).

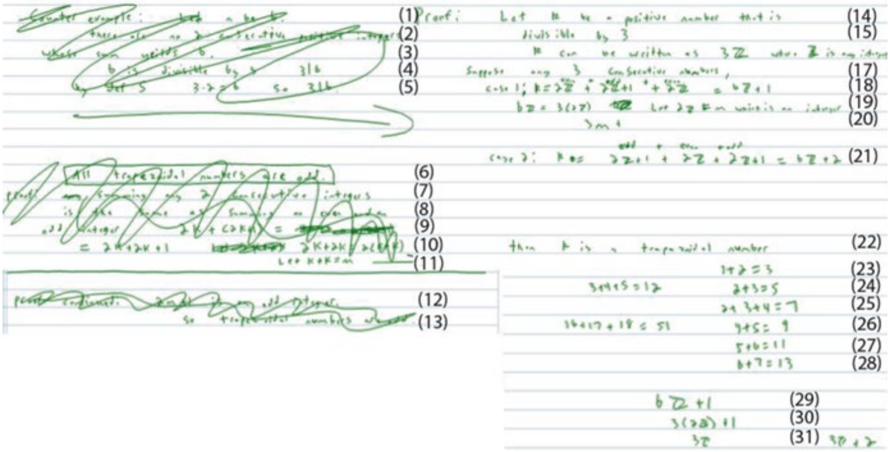

Fig. 3.7 Proving attempts of Marty on the trapezoid theorem

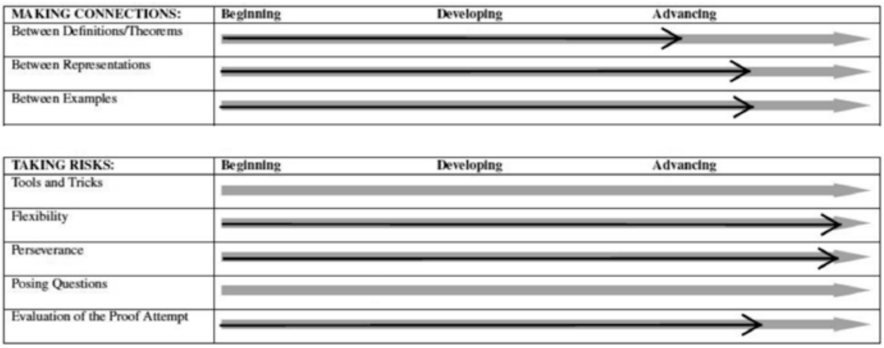

Fig. 3.8 Levels of Marty's work on the trapezoid theorem

Marty's proving attempts for the trapezoid theorem are located in Fig. 3.7. The levels coded for Marty are in Fig. 3.8.

Note that Marty produced examples in his proving (Lines 23–31), and used those examples to find the key idea (lines 18 and 21). We did not see any evidence of the subcategories *Tools and Tricks* and *Posing Questions*. Also, Marty's work was placed in "advancing" levels for many of the other subcategories, despite the production of an incorrect proof.

Overall, we noticed that Marty mostly demonstrated "beginning" level actions on both categories (Making Connections and Taking Risks) for the digit theorem (see Fig. 3.4) with the exception of no level for the subcategory *Tools and Tricks* of the Taking Risks category. As shown in Fig. 3.8, he had mostly "advancing" level actions on both categories (Making Connections and Taking Risks) for his work on the trapezoid theorem, with the exception of no levels for subcategories, Tools and

Tricks and Posing Questions. Greg, on the other hand, had “advancing” level actions for the Making Connection subcategories and mostly “developing” for the Taking Risks for the digit theorem (see Fig. 3.2), with the exception of no level for subcategory Posing Questions. For the trapezoid theorem, Greg’s work had varying levels of actions for subcategories: mostly “developing” for Making Connections, and varying levels between mid- “developing” to high- “advancing” for Taking Risks (see Fig. 3.6).

3.5 Discussion

In this section, we delineate some observations and provide hypotheses that were generated from the case studies in the previous section. Specifically, we highlight the following: (i) the similarity between the coded levels of some categories between the two students; (ii) the connection between students’ work and their perspectives on creativity; (iii) the fact that correctness was not taken into consideration while coding; and (iv) the association between some of the subcategories in the rubric. In addition, we suggest some teaching practices that complement the CPR on Proving.

3.5.1 Remarks on the Coding of Students’ Work

The coding of Greg and Marty’s work illustrates the evaluation of the whole proving process rather than the final product. This analysis highlighted how two students’ different proving processes on the same task could be coded at the same level for a subcategory. For example, both students’ differing processes were coded “advancing” for the subcategory *Flexibility* on the trapezoid theorem, but there was variation in the number of proof attempts and approaches between Greg and Marty. We developed our rubric intentionally to capture such instances to stress the valuing of assessing individual student’s work.

Our second observation is that both students’ coded work had some alignment with their perceived notion of creativity as they shared it in the exit interview conducted at the end of the semester. For example, Greg stated that mathematical creativity is “coming up with the little tricks that make each proof flow.” He utilized modular arithmetic (Lines 34 and 36 in Fig. 3.1), which was unconventional for the course, and thus we considered it a “trick.” Marty, on the other hand, defined mathematical creativity as “start[ing] from a different place or us[ing] a different method. You know, induction, contradiction, all those sorts of things.” This description was observed in his proving attempts of the trapezoid theorem, since he first attempted to disprove this task using a counterexample, then shifted his approach to two different direct proofs (Fig. 3.7), thereby demonstrating an “advancing” level of Flexibility. This possible link between students’ perceived notion of creativity and their proof attempts requires further investigation. However, we hypothesize that students' use of the CPR on their proof attempts could potentially help them to reach their creative potentials by illuminating other aspects of the proving process that students may not have focused on.

In our data, incorrect final proofs did not associate with less potential for mathematical creativity. Marty's work for the trapezoid theorem is an incorrect proof, however many of his actions in his proving attempts were coded in the Taking Risks category as "advancing." The CPR on Proving was designed to purposefully remove judgment of validity in order to encourage students to take risks and to be engaged in the proving process. Also, it is possible that some students can be relatively creative while being incorrect, which is a sentiment shared by one of our interviewed mathematicians, Dr. F:

> I will risk it and say that [a proof] doesn't have to be correct to be creative. But at least it [the proof] should be fixable. It can happen that you have an original idea and you mess up details, which is not surprising because if it is an original idea then it means that you haven't practiced that, [so] you would make mistakes.

Some subcategories can help develop or enhance other subcategories in a student's proving process. Greg demonstrated "advancing" level actions in the *Tools and Tricks* and *Perseverance* subcategories of Taking Risks for the digit theorem (Fig. 3.2). In the coding analysis, we noticed that his evaluation actions for each of his individual attempts combined with his perseverance contributed to his overall "developing" level of *Flexibility* on this task. Similarly, our analysis of Marty's different attempts indicated that his process of evaluation of each attempt and his overall perseverance allowed him to demonstrate advancing level actions of flexibility. Noticing such overlaps with the subcategory of *Perseverance,* and subcategories *Flexibility* and *Evaluation*, we revised the category of Taking Risks and eliminated *Perseverance* subcategory (for details see Karakok et al. 2016). It is also possible to think there are overlaps between the categories of Making Connections and Taking Risks. For instance, Greg took a risk to employ a trick from a previous course to prove the digit theorem, thereby making a connection. However, these subcategories include unique elements and actions which help distinguish them from each other, such as utilization of a "trick" in Greg's example. The user can leverage those unique elements to improve or enhance his/her proving process.

We believe that it is possible for a student to engage in proving attempts that do not necessarily exhibit all advancing-level actions but still demonstrate relative creativity. Reflection on each subcategory of the CPR on Proving can help develop a student's creative potential for proving. For instance, a student might approach a proof with one technique (a "beginning" level in *Flexibility*), without creating any Tools and Tricks, but persevere in the process at the advanced level by creating many examples and generalizing them to gain an understanding of the key idea of the proof (advancing level in *Between Examples*). Another example comes from Marty, his work did not have any evidence of actions to be coded in the *Tools and Tricks* subcategory, but showed the potential of creativity in his proving.

3.5.2 Teaching Implications

The CPR on Proving was created for instructors' or students' use. For each subcategory, instructors can evaluate their students' proving attempts and decide which part of the continuum the work could be placed. Once trained on how to use the rubric, students can also do this for their own and also for their peers' work. Our intention is that if an instructor or a student evaluates proving attempts, s/he can hopefully see what needs to be improved or worked on during future tasks (attempts).

We have found that the use of the CPR on Proving may not elicit all of the subcategories for certain tasks. For example, the rubric may not be very useful for either a student or instructor in examining work on a routine task or typical exercise (as opposed to Schoenfeld's (1982) definition of a problem). However, a task that can be proved using several different proof techniques can challenge students to be creative in their explorations during their proof attempts. Both theorems used in the case studies are examples of such tasks (e.g., Zazkis and Holton 2009; Leikin 2014). The use of CPR on Proving could also help in the scenario of "getting stuck" or encountering a proving impasse (Savic 2015), since most students "[produce] either no solution, incomplete solutions, or the 'standard' ones" (Zazkis and Holton 2009, p. 349).

Mathematicians such as Pólya (1954) have noted the importance of guessing theorems as a creative endeavor. Therefore, we suggest engaging students in the process of creating conjectures, posing problems, or solving open-ended tasks (Silver 1997; Brown and Walter 2005) with the CPR on Proving. Asking students to conjecture can challenge them to create examples in order to generalize, can encourage them to ask questions about their conjectures, or to evaluate the key ideas that make their conjectures true or false. Use of the CPR on Proving while students are engaged in these tasks may both improve their proving process and increase their potential for mathematical creativity.

The CPR on Proving provides an opportunity for teachers to be informed on students' proving processes. However, it is crucial that the instructor does not use the rubric to label or categorize students' level of creativity in broader generality. This might cause students to feel less creative, and perhaps consequently do poorly in a proof course. We recommend an open discussion on the rubric between the instructor and the students early on to unpack the meaning of each category, subcategory and underlying levels, and demonstrate the usage of the rubric. Some suggestions might include an instructor using the CPR on Proving after a student demonstrates his/her proof attempt in class, or having students discuss their own creative proof attempts using laminate copies of the rubric (for re-use) with dry-erase markers. Also, the CPR does not have to be utilized in its entirety. It might be appropriate for an instructor or a student to focus on one subcategory in order to highlight certain areas of improvement. For example, it might be useful for the students to examine an unsuccessful or successful proof and ask them to reflect on the key ideas that made it so. This would help them in the *Evaluation of the Proof Attempt*.

Lastly, we suggest creating an environment in the classroom where creativity can be nurtured. This includes allowing students to make mistakes, perhaps even valuing and discussing them in class. In order to successfully achieve this, students' grades may not be penalized as frequently for making mistakes (see Burger and Starbird (2012) for similar suggestions). Instructors could be explicit about the tasks for which they expect students to provide a correct proof and those for which they expect students to explore freely without judging correctness. Incorrectness, as we have seen in both Greg and Marty's work, and as Dr. F stated above, can be a catalyst for creativity.

3.6 Conclusion

The development, and subsequent implementation, of our rubric is meant to start a discussion to encourage and promote mathematical creativity in the tertiary level. We do not claim that this rubric encompasses all the behaviors that can lead to mathematical creativity. Furthermore, we recognize that there may be other subcategories, so it is not our intention to limit the discussion of the process of creativity with these subcategories. The intention of the CPR on Proving is to make the behaviors that mathematicians themselves exhibit during their pursuit of new ideas explicit to the students.

As a research group, our overall goal for the Creativity-in-Progress Rubric (CPR) on Proving is to help tertiary students reflect on their proving processes, and hopefully, alleviate potential proving difficulties that are common to most students at the post-secondary level. Guilford (1975) highlighted the importance of reflection, stating that "the student [should] be taught about the nature of his[/her] own intellectual resources, so that [s/]he may gain more control over them" (p. 120). The function of the rubric, as we see it, is not to determine students' exact levels of creativity, nor to state that one student is or is not creative. Instead, our rubric is about fostering growth. It is about encouraging students to engage in behaviors that mathematicians claimed may lead to mathematical creativity. We believe crucial actions that can lead to the potential for mathematical creativity in proving are embedded in the advancing levels of the rubric. Regardless of the school level of the student:

> It must not be forgotten that the basic law of children's creativity is that its value lies not in its results, not in the product of creation, but in the process itself. It is not important what children create, but that they do create, that they exercise and implement their creative imagination. (Vygotsky 2004, p. 72)

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