Researching in Undergraduate Mathematics Education: Possible Directions for both Undergraduate Students and Faculty

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Abstract Research in Undergraduate Mathematics Education (RUME) is a new field to both mathematics and mathematics education. It borrows theory and methodology from other disciplines including psychology, sociology, and neurology. At its core, RUME is attempting to find out about the teaching and learning of undergraduate mathematics education in order to improve it. In this book chapter, I attempt to give a quick overview on how to conduct RUME with undergraduate students. I pull from my experiences as a mentor of ten undergraduate projects. There is also a suggested timeline of RUME in a semester, some ways to generate RUME open questions, and a large amount of open questions conjectured by others. My hope is that this book chapter has information for both mentors and undergraduates alike.

Suggested prerequisites. *For research in undergraduate mathematics education, a pre-requisite may be the mathematical knowledge of whatever topic you would like to research. For example, if an undergraduate student wants to research in the teaching of real analysis, they must have some knowledge of real analysis topics in order to understand the mathematics in the education. The rest of the preparation before research can be accomplished during a semester or year.*

1 Introduction

Research in undergraduate mathematics education (RUME) is a relatively new field, with researchers beginning to publish and discuss findings in the 1980's [23]. Much of the work in RUME is focused on humans at the undergraduate level and how they teach or learn mathematics. RUME has links to many other disciplines, whether it be psychology, neurology, social sciences, ethnography, or (and perhaps most importantly) mathematics education at the K-12 level. Therefore, the possibilities of an undergraduate researching in RUME are endless.

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In this chapter, I detail what *I* deem is necessary and sufficient for undergraduate research in RUME and offer both open questions and ways to generate more. The audience of this book chapter is new(ish) faculty that have students interested in RUME. One caveat is that the faculty themselves must be somewhat comfortable with RUME or be open to learning new RUME topics. For example, I have mentored ten undergraduate students in research of some type, and every mentoring process I have learned new ways of approaching research with undergraduate students. However, I think that I would have liked an outline or more suggestions to mentoring than the word-of-mouth or experiences I received as a graduate student. This book chapter is an aggregation of those first-hand experiences. Another audience is undergraduate students themselves. I hope that an undergraduate can pick up some ideas about how to generate research questions or common structure of a thesis from this chapter. In fact, section 4 is written directly for undergraduate students.

This book chapter will not include everything on RUME. It is anecdotal in nature. Many of the citations here are ones that have been influential for me as a student and a researcher. For another approach to RUME, please see what Selden and Selden [24] have written. Intrinsic to data analysis is the filtering of what the researcher attends to and how they make sense of what they see. Some of these theoretical perspectives are also presented by Selden and Selden. Here, I offer a cursory view of theoretical perspectives for two reasons: one is that I am gaining knowledge about theoretical perspectives every year, but do not feel expert enough to explain each or all of the perspectives. The second reason is with the short timeline that an undergraduate student has, I have often guided them towards more open questions that can cite articles which utilize other RUME theoretical perspectives (see [24] for more on this subject).

2 Beginning RUME With a Question

There are possibly two routes that a faculty member could go with undergraduates who want to do RUME. The first route is to incorporate the student in to one of the faculty member's projects. This route has some positives and negatives for the student and the research supervisor. On the one hand, there is likely a developed research plan already in place. This may include a research proposal, a collection of related research literature, and perhaps data that has already been collected. Thus, when bringing a student into an existing project, there is a pretty clear path to follow: familiarize the student with the research plan, have the student read the related research literature, and then have them reflect on that literature (and possibly the already collected data) to develop a research question that fits into the existing project. On the other hand, however, this route may not offer the student the same level of motivation since they are assuming another person's project. It is for this reason alone that I have yet to go this route for my own mentorship, although I can recognize how much time and effort would have been reduced by incorporating an undergraduate into my own projects.

The second route is to have the student generate the question. As an undergraduate student at the University of Oklahoma, Katherine (Kaki) Simmons approached me about doing a directed research project. I think she assumed that she was going to join me on my research, but I asked her what she was interested in. She stated that she started an American Sign Language club at the university and was interested in Deaf and hard-of-hearing (D/HH) students' experiences in undergraduate mathematics courses. Therefore, I asked her to look at the previous literature for this intersection of D/HH and undergraduate mathematics education. Kaki did not find much literature in undergraduate math education, but did find some K-12 math education literature about D/HH students. So her research question ended up being: "What roadblocks and successes happened in D/HH students' undergraduate mathematics education?" While being an example of a first encounter with advising an undergraduate student, this is also an example of how to generate a RUME open question.

Therefore, if a student emails me with a request to research, I ask the question, "Why RUME? What part of RUME interests you, either as a student or a future teacher?" I believe having the student ask the research question first may allow or sustain intrinsic motivation [19] that the student has for the study (see 4.1 for more). In this same first meeting, I show how I would approach research in a certain topic, including utilizing Google Scholar and other internet resources. I may also bring up a couple of foundational articles or learning researchers and show them how to do "reverse citations". Describing a research question into keywords to search is critical because it can be difficult in RUME to have a synonymous vocabulary about teaching or learning. This might be due to the field of RUME being so "young." I suggest articulating and being specific about one's research question.

Critical thinking about an article can also be a way of generating new research questions. For example, a research question of mine came from a reading of Sio and Ormerod [25], where the authors stated that all 117 psychological studies analyzed included an incubation period in their study from 1-60 minutes. An incubation period is a break in problem solving, which precedes an AHA! moment [12]. I kept saying to myself, "When I do math proofs in graduate school, I usually take days to incubate and think about approaches." So, I attempted to investigate what I felt was incubation in mathematics [21] using methods that could allow for incubation over a few days or weeks. It is up to the advisor to figure out how to mold an undergraduate's research question into something "new" for the field. There are a large amount of suggestions for research questions towards the end of this book chapter (see section 6).

After having a meeting with the student, I always suggest that they take two weeks and try to read quite a bit about their interest. Reading previous research has multiple benefits for undergraduate students:

1. **Students will need to situate their research within the field's work for any publication.** Whether their research question is extremely novel or well-researched, more reading of a similar topic builds more knowledge of the subject.
2. **Reading helps refine research questions.** By reading many articles on the topic of interest, students may find that their research question has been explored in

previous studies. Also, the student can attempt to position their question as different from those same previous studies.

3. **Reading provides models of frameworks, methods, and interview tasks that can be used.** Some studies provide the exact surveys or questions in appendices, and those methodological tools can assist in their research. If one aspect of the research question is more developed, borrowing methods is very good for establishing their perspective on the study (see 4.2 for more).
4. **Reading is important for examples of conducting and writing research.** A student can see a template and examples of what RUME research looks like (see 4.5.1 for a suggested template). The background literature sections of any article, in particular, could be useful to find more literature on the same or a similar subject.

Once you've conjectured a research question and started to read some research, then one must figure out which theoretical way can one answer the question. For example, suppose that one wants to research how students learn partial derivatives. There are many perspectives on how students learn a concept: is it their engagement with the material, or their peers, or what beliefs arise with learning partial derivatives? These different perspectives require different data collection techniques, and each one of these perspectives is associated to some educational (or other) theory.

In RUME, one usually "grounds" their work in a philosophical theory about education. This is similar to mathematics research; one can assume the axiom of choice or not, and because a mathematician assumed an axiom, it influences how they look at or create their subsequent mathematics. Influential authors of educational psychology theory include Piaget, Vygotsky, and Bandura (for an overview, please see [30]). These theories may sometimes be difficult to understand, so if you do include a theoretical background with undergraduates, please approach this with thoughtfulness and care. These perspectives usually align with certain methodologies. In the example above, if I were mentoring an undergraduate on investigating the beliefs of students as they pertain to partial derivatives, I would suggest reading up on Pajares [18], belief and affect theory (e.g., [8]), and any article about students and partial derivatives (e.g., [7] which is in physics education). The reading, which is already situated in a theoretical perspective, now allows me to figure out which method of research I should suggest.

3 Common Methods of RUME Research

There are many ways of approaching an undergraduate mathematics education research study. In this section, I explain some of the common methods that I have seen. I also detail some procedures of each method, and attempt to give ample examples of the RUME research that influenced me in that method.

3.1 Qualitative Research

Qualitative research [1] is another method of conducting RUME, and one that I personally use most often with undergraduate students due to my own comfort, sample sizes, and time. I believe the reason why one would use qualitative methods is to explore the *why* and *how* of certain mathematics education phenomena. It is to "dig deeper" into interesting situations. For example, Ellis, Fosdick, and Rasmussen [6] concluded, using quantitative data, that women are 1.5 times more likely to leave STEM majors after Calculus compared to men. However, this result generates more research questions, including "*why* are women leaving?". Ellis and Cooper [5], using open-ended prompts in the quantitative study and qualitative methods, attempted to answer this question. They coded 454 open-ended responses and found "the proportion of affective statements made by male and female Switchers [out of a STEM major] was significantly greater than the proportion for Persisters [staying in a STEM major]" [5, p. 133], and also that females specifically reported lower self-efficacy, which may come from more negative experiences with the instructor.

Data can come from many other sources. One may want to answer a research question about teaching ("How does a teacher introduce the Fundamental Theorem of Calculus?"), so data collected may be video observations of multiple teachers teaching the theorem. Depending on the research question, other data collection can include written work or video.

When done with the data collection, data analysis is another aspect of qualitative research that researchers must be careful to conduct and document. For example, one could take all the videos for recording the Fundamental Theorem of Calculus and rewatch them to look for actions that the teacher conducted during the class period. Then, after cataloging the actions, the researcher could categorize the actions into larger groups that could describe many actions. Saldaña [20] has a wonderful resource for coding qualitative data which explains what all the qualitative coding techniques are and in what situations one might use them. When mentoring students in qualitative research, I have them read excerpts from this handbook that I believe will suit their needs.

Larnell [10] is an article that has some of the most robust coding details I have seen, and is, in my mind, one of the most influential qualitative articles in our field. He observed 10 weeks of a course, wrote field notes, audio-recorded post-observation notes, and conducted 5-6 semi-structured interviews of 1-2 hours each with two participants. The author then went through two phases of identity coding with two participants, Vanessa and Cedric, and re-analyzed for a "confluence of themes... or the confluence of socialization forces and themes" [10, p. 246]. Larnell concluded that "the institution itself can present barriers to Black learners and their identities" and that the two students are constantly "negotiating" their social, racial, and mathematical identities [10, p. 261]. The author ended the article with appendices that detailed the coding he did for sample data.

[1] A broad scope of definitions of qualitative research can be found in Chesebro and Borisoff [2].

One of the most common ways that qualitative research is conducted is through clinical interviews. However, there are many other different methods, including conducting a literature review, teaching experiment, or textbook analysis. I detail each of these below.

3.1.1 Literature Review

A literature review is usually qualitative research where the author tries to aggregate the books, articles, and any other published items of a certain topic in order to find common themes. For example, Leyva [11] did a general search on sex and gender in mathematics education on Google scholar. The author then filtered through "peer-reviewed articles published in the top 100 journals across the fields of education, mathematics (with a focus on education), and gender studies" [11, p. 399]. Finally, Leyva reduced the amount of articles to 56 based on citation amounts and years published. With those 56 articles, Leyva found that there were mainly "two perspectives on studying gender in mathematics education - namely, achievement and participation" [11, p. 425], while also arguing for more detailed research on gender in the form of intersectionality theory.

Literature reviews do not required an Institutional Review Board application, since there are no human subjects in this research. This might be an advantage for some undergraduates, especially those that do not much time to collect data for qualitative or quantitative research. However, literature reviews require use of time differently; a critical reading of the material is required, as well as deep thought about the themes and ideas that emerge.

3.1.2 Teaching Experiment

A teaching experiment is qualitative research where a researcher recruits a handful of students and sets up a series of tasks. These tasks and meetings are separate from their enrolled courses. While doing these tasks, students' mathematics emerges, and the researcher is the person that constructs models of those emergent actions. Swinyard and Lockwood [28, p. 11] stated that "the researcher's central purpose in a teaching experiment is to construct a model of student thinking or reasoning in relation to a particular concept or idea." The authors cited Steffe and Thompson [27] who wrote an essential treatise on why and how to conduct a teaching experiment. For example, Cook [4] examined how two students came to understand properties of algebraic rings through four sessions each lasting 75-90 minutes long and centered around solving equations. One of the results is that utilizing secondary algebra is a starting point can be effective if student thinking can be leveraged productively.

3.1.3 Textbook/Material Analysis

A textbook or material analysis is one method of doing RUME that can be a good way of incorporating undergraduates into research. It also requires no human subjects, so there is no Institutional Review Board application needed (see 4.3). What it frequently entails is an examination of the constructs or communication of some materials, often with quantitative variables or statistics. One could ask: "What are the kinds of math problems posed written in a textbook?" For example, Lithner [13] examined the exercises in a Calculus textbook using three constructs of how a student could approach the exercise: *identification of similarities* or solving using earlier methods described in the book in solved examples, *local plausible reasoning* or slight modifications in problem solving from what has been previously done in the book, and *global plausible reasoning* or analyzing and considering intrinsic mathematical properties of the components in the exercise. Lithner concluded that this textbook had 85% of the exercises as identification of similarities. Mkhatshwa and Doerr [16] did a similar study on business calculus textbooks.

Tallman and Carlson [29, p. 105] investigated 150 post-secondary Calculus I exams and found that "exams generally require low levels of cognitive demand, seldom contain problems stated in a real-world context, rarely elicit explanation, and do not require students to demonstrate or apply their understanding of the course's central ideas." One critical thinking aspect to consider is based off of Lithner [13]: what beliefs of mathematics are the students acquiring by looking at textbook (or exam) tasks? This question seems to be important especially for undergraduates that would like to be in graduate school or other industry leadership roles, since they may be in charge of training or teaching others.

3.2 Quantitative Research

From many qualitative studies, categories are either used or created. Then, one can either discuss what the categories mean for undergraduate learning and teaching, relationships between the categories, or create a new framework that encapsulates many of the categories. There is another approach to use the categories; with any of these approaches and codings, frequencies appear and quantitative research can be conducted in order to discuss results or answer research questions further.

Quantitative research, particularly for undergraduate students, usually entails accumulating data through surveys or other metrics and using statistics to answer research questions. For example, a student of mine, Ben Gochanour, wanted to investigate the relationship between mathematics anxiety and mathematical motivation, since both constructs have been primarily investigated with mathematical performance (i.e., grades or test scores). He created an online survey with both math anxiety and math motivation questions from previous surveys [14, 9]. The subsequent validation, correlation, and/or causation statistics studied of that online survey would be examples of questions to answer using quantitative research.

Statistical measures are common, so a large sample size would be important. Also, a recommendation would be to have the student take a statistics course or be very fluent with some statistical software (i.e., R or SPSS). Examples of quantitative studies include the previously discussed Ellis, Fosdick, and Rasmussen [6] article as well as Melhuish [15]. Melhuish took many articles on group theory in abstract algebra that had small-scale qualitative studies and replicated them quantitatively. Her results showed that replication with large-scale sample sizes can "validate theories by reproducing a number of the original results...[using] the general population of undergraduate students taking an introductory abstract algebra course in the United States" [15, p. 31].

One of the biggest questions with a quantitative study is one of reason of correlation or causation. For example, why did some statistically significant result happen with the population one was studying? Making explicit the discussion of statistical results and limitations will go a long way to helping undergraduate students be critical and mindful for their future work.

Mixed-methods research maybe be another (perhaps strong) approach to research since it combines the quantitative and qualitative methods. It gives both perspectives of research and can answer questions with more information than one method alone. However, mixed methods research can be time-consuming, so I would personally suggest that undergraduates may be guided to one method or another.

4 A sample timeline for mentoring undergraduate students in RUME

This section is concerned with a (non-linear) timeline for RUME research that I have experienced with undergraduates. Again, some of this will vary due to the choice of the mentor, as well as if the undergraduate is being incorporated into an existing research project.

4.1 Research Question

A research question is much like a conjecture in mathematics. One is interested with a self-observed phenomena and wants to see if this is a commonality. Some RUME researchers have undergraduate students work on projects that have been already started, and the research question is a sub-investigation of the larger study. Others let the student generate a research question and negotiate how one could research in that topic. Understandably, both approaches have limits, especially due to the amount of time and energy of both parties, along with the professor's background on methods or topics. There are many research questions aggregated in section 6.

4.2 Background Literature

An undergraduate student comes to you with the research question. What do you do? First, an examination of what has been done previously would be a good step. Perhaps the research question has been already examined, but not in the setting or environment that the undergraduate student has proposed. Some institutions require undergraduate students to have at least five citations before they even submit an Institutional Review Board (IRB) application (see 4.3). Reading literature is in itself a very difficult task[2]. Mentors should read some literature with the student and discuss in order to reach some mutual understanding of the topics. Much like mathematics, mathematics education has terms and definitions. However, as Selden and Selden [24, p. 432] stated, "Definitions do not have, perhaps cannot have, mathematical precision. Concepts may only be approachable rather than precisely definable." Some literature not only has definitions but the methodological approaches for how they're examining those definitions, and one can look deeper into those methodologies in order to figure out their own. One suggestion is to also properly define each of the terms used in the student's research question. This will often bring up theoretical constructs that need to be addressed.

4.3 Institutional Review Board Application (if needed)

Almost every institution has an institutional review board (IRB). This review board makes sure that any human-based research projects are within the bounds and scopes of ethics. Prior to applying for permission to research using human subjects, each researcher must be trained in ethics and compliance. This is usually done online through the Collaborative Institutional Training Initiative program [3], and linked to your university. One must plan ahead for this process; the CITI training and the IRB process can both be lengthy. If your project is to be completed in one semester, and the data collected has to also be conducted in the same semester, I would suggest to start writing pieces of the article or thesis that can be done while you wait for the approval.

4.4 Data Collection and Analysis

If you are using human subjects in your research, once you've received permission from your IRB, you can start the data-collection process. Depending on the research conducted, this could take up to one month. The data collection should be in accordance with the methodology that you are choosing.

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[2] How one reads mathematics education literature is, in itself, an open RUME question!

Data analysis is also different depending on methodology chosen, so make sure that you check multiple sources about data analysis before performing it. One of the ways to alleviate difficulties in writing is to document, either any journal or set of notes, your data collection and data analysis. This way, when you get to the writing process some aspects of the thesis are going to be easier to write out.

4.5 Writing or Presenting Findings

Writing and/or presenting research might be the most difficult aspect of the research process. Once data analysis has ended, one has to interpret what the results mean. This may involve more questions than answers. In this subsection, I will detail the aspects of a typical RUME paper/article/thesis. I will write a bit about what each section includes in order to give guidance. There is no requirement for any of these sections, and authors may combine sections together if appropriate.

4.5.1 Typical RUME paper sections

1. **Introduction**. An introduction is typically where one introduces the setting for the research. This may involve stating the math topic or motivation to examine a certain math process. It may also be a place to state what sections are coming in the paper, along with a short description of what each section will discuss.
2. **Background Literature**. This section should be where the majority of your citations of previous research should be. A thorough background literature section should provide both what has been done in the area of the research question while also providing rationale for why your paper is adding to or filling in the previous research. My advice is that a background literature section, in both a paper and a presentation, should be a communication to the audience that you have done your "homework" as well as you could. When doing my dissertation, I had difficulty with not knowing if there were any journal articles that I was missing. Over time, I have been better with my conscience about being fine with not knowing all of the literature. I also know that there are reviewers that will cite other literature that I might have missed.
3. **Theoretical Framework**. This section may be included if one has a theoretical background that they are coming from. Please see section 2 or [24] for more details.
4. **Research Question(s)**. Here is where one writes the question(s) that they wanted to investigate. Many of the previous sections build up to this inquiry, and the rest of the article or thesis deals with how to (try to) answer this question(s).
5. **Methods for Data Collection and Analysis**. I would suggest that this section be as detailed as possible. I also suggest that students include their timeline of both data collection and analysis. If these details are available, please include demographic data with your participants. If possible, and if the participants did

not provide their own pseudonyms, I would suggest pseudonyms that encode important information. An example of this pseudonym choice is in section 5.

6. **Results**. The results section is where one presents the data collected or results found. I believe this section should be with as little interpretation or opinion as possible, although some authors combine their results and discussion together. The advice I give to students is to just present the data in the results section.
7. **Discussion**. Here is where the author can discuss and offer the interpretation for the previous section. This is where interpretation and conjecture can happen for the author. It can also be the section that addresses the question, "So what?" or "Why does this research matter?"
8. **Conclusion**. The conclusion should recap the whole article. I believe that we, as humans, only remember a few ideas (for lack of a better word) for any interaction, including reading articles. Therefore, I usually advise that authors include the two big ideas, results, or thoughts that any reader should come away from this article in the conclusion.

5 Paxton's Undergraduate Research Experience

For this section, I will give an example of an experience I had with an honors student, Paxton Martin Clark, doing her thesis in RUME [22][3]. All of the research, including reading, IRB, data collection, and writing was done over winter and the spring semester.

Paxton was a pre-med as well as a math major, and she knew that calculus was a requirement for her and many of her peers in pre-med. This requirement of calculus was brought up in her search of articles that mention pre-med and mathematics [17]. However, she also recognized that there was not much literature about pre-med and math. In order to take a first step in this intersection of fields, her (purposely vague) research question was: "what are pre-med majors and physicians' perceptions of undergraduate mathematics education, and what are differences of perceptions between the two groups?"

For Paxton's study, she wanted to know about the intersection of pre-med students and mathematics education. One article by Nusbaum [17] about levels of calculus and pre-med students was the start of the research, and the next step was to see what this article cited or what authors cited Nusbaum's article. We also looked at pre-nursing or nursing students and mathematics [1], since there was little pre-med and math literature. We did not gain much in terms of methodology and opted instead to do semi-structured interviews with an open qualitative analysis (see 4.4).

Paxton went through the CITI training immediately after her initial meeting with me. This meeting was arranged in Fall, before her honors thesis work had begun in the Spring. Once done with CITI training, she and I discussed her research question. Since there was little in the literature about this research question, we decided

[3] After Paxton submitted her honors thesis, I was so overwhelmed by both the novelty and results of her study that I offered to help her restructure, rewrite, and publish the paper.

on doing a qualitative study in order to gather more information about mathematics pertaining to Pre-Medicine students and physicians. We then discussed what questions she would like to ask both sets of participants and figure out how we would recruit each set of participants. In our IRB application, we were required to detail all of the processes and all of the questions that we used with the participants. This included how we collected the data and what we did with the original data. We also had to make a consent form that allowed the participants to see what the experiment is all about and give their permission to participate.

For example, in Paxton's study, she did a qualitative study that involved interviewing six participants. Each interview was audio-recorded and transcribed, and she gave each student a pseudonym S1-S6 while the physicians were PH1-PH3. Paxton used a process of coding and categorizing, where she initially coded based upon the answers that the interviewees gave and gathered the codes into categories (see [20] and 3.1 for details). In her results section, she reported on what the participants said in their interviews, presenting the interview data in the same categories she created after her second coding. For example, in her thesis, she gathered quotes into a section labeled "Technological Advances and how that affects math," and one of the quotes that she presented was from a practicing physician talking about more complicated mathematics: "It's actually still pretty integral. I don't personally do the calculations, but I review what the computers say. I have to make sure that they do make sense. The computers help you, and aid you, but they don't replace you." More quotes from the participants were cited in her thesis under that 'technology" category.

Paxton, in her thesis, offered what she believed were some reasons for why the physicians and the pre-med students differed in their beliefs about the relevance of undergraduate mathematics education. She finished her thesis by conjecturing how mathematics educators may use the categories from the interview data in their classrooms.

6 Suggested Projects

Since RUME uses many human-based research questions, and RUME is relatively young, there are infinite ways to generate research questions in RUME. When an undergraduate approaches with an interest in researching with me, the first question I have for them is what *they* are interested when it comes to teaching and learning mathematics. The second question is what they think that they will want to do after this research project; I would steer them towards some research plan that they will enjoy semi-intrinsically.

Perhaps in order to motivate readers interested in undergraduate students and RUME, I offer two collections of open questions in RUME. In 2016, Gulden Karakok (University of Northern Colorado) and I hosted a conference, RUME with a View, that was supposed to be for fairly new researchers to RUME to generate open questions in RUME. 93 participants gathered into five breakout groups in order to both situate themselves with the literature and conjecture questions for the future of RUME to research. Some questions that the participants of the conference generated are listed below:

Research Project 1. Questions generated by the RUME with a View Conference participants

- What does students' mathematical journal entries, notebooks, assignments (or written work), and specifically their use of language, reveal about their mathematical understanding?
- How do students' mathematical identities (or affect generally) influence their engagement in pre/business/regular calculus?
- How does collaboration affect the ability to mathematical problem solving or the mathematical problem-solving process?
- What forms do microaggressions take in undergraduate mathematics classrooms? How do students perceive these microaggressions? And, what are the impacts on the students' sense of belongingness?
- What do introductory statistics courses look like across the country at different schools? Why are they offered (for different purposes) and what topics are discussed?

There was also a recent literature review (see 4.2) on content topics that were under-researched in RUME. This was conducted by Speer and Kung [26], who wanted to investigate the "complement" of RUME research. They [26, p. 1292] stated that:

> Knowing where theory development and findings are scarce or plentiful can help researchers (and those who advise them) to know whether their chosen topic is apt to take them into well-understood territory or whether they will encounter few studies and perhaps only limited theoretical frameworks to guide their efforts.

The authors created a list of topics in calculus that have and have not been researched. I cite the topics that have not been researched below:

Research Project 2. Content topics in calculus (from Speer and Kung [26, pp.1291–1292])

- implicit differentiation, in particular examinations of what sense students make in the transition from df/dx to d/dx and the idea of differentiation as an operator;
- student thinking and sense making about linear approximation and differentials;

- connections between trigonometric functions (as ratios of lengths of triangle sides) and the calculus of them;
- Newton's method, in particular what sense students make of the process.
- integration techniques involving the very commonly-used method of substitution;
- other integration techniques and what sense, if any, students make of this topic;
- volumes of revolution
- power series, especially the question of what sense students make of the overarching idea of approximating one function with other functions;
- Taylor and Maclaurin Series and what students think the core ideas are behind the computations we ask of them.

7 Conclusion

RUME is a difficult endeavor for anyone. There are many aspects of research that even I am still learning about. Therefore, when an undergraduate student embarks on a semester (or year(s)) journey of RUME, I believe that constant reflection of the journey is necessary. Going through CITI training, then an IRB proposal, then conducting the data collection, then analysis, and then writing and presenting the research is gigantic. In this book chapter, I've tried to make this process explicit for new RUME researchers.

More often than not, what an undergraduate researches in RUME is new to the field. Therefore, I want mentors and mentees to take stock of what they've done throughout their research. I suggest mentors constantly reflect on their advice and how they have nurtured student growth. I suggest mentees look back at how much they've accomplished themselves. I truly believe research by undergraduates is a formative experience that may ultimately change their lives.

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