PEDAGOGICAL PRACTICES THAT FOSTER MATHEMATICAL CREATIVITY AT TERTIARY-LEVEL PROOF-BASED COURSES

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MATHEMATICS
• Creativity-in-Progress Rubric on Proving (Savic et al., 2016; Karakok et al., 2015, 2016; El Turkey et al., under revision)

• Two categories:
  • Taking risks
  • Making Connections

• First created as a research tool, but has morphed into a tool for teacher and student use
## CPR on Proving

### MAKING CONNECTIONS:

<table>
<thead>
<tr>
<th></th>
<th>Beginning</th>
<th>Developing</th>
<th>Advancing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Definitions/Theorems</td>
<td>Recognizes some relevant definitions/theorems from the course or textbook with no attempts to connect them in their proving</td>
<td>Recognizes some relevant definitions/theorems from the course and attempts to connect them in their proving</td>
<td>Implements relevant definitions/theorems from the course and/or other resources outside the course in their proving</td>
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<tr>
<td>Between Representations(^1)</td>
<td>Provides a representation with no attempts to connect it to another representation</td>
<td>Provides multiple representations and recognizes connections between representations</td>
<td>Provides multiple representations and uses connections between different representations</td>
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<tr>
<td>Between Examples</td>
<td>Generates one or two specific examples with no attempt to connect them</td>
<td>Generates one or two specific examples and recognizes a connection between them</td>
<td>Generates several specific examples and uses the key idea synthesized from their generation</td>
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## CPR on Proving

### TAKING RISKS:

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<tr>
<td><strong>Tools and Tricks</strong></td>
<td>Uses a tool or trick that is algorithmic or conventional for the course or the student</td>
<td>Uses a tool or trick that is model-based or partly unconventional for the course or the student</td>
<td>Creates a tool or trick that is unconventional for the course or the student</td>
</tr>
<tr>
<td><strong>Flexibility</strong></td>
<td>Begins a proof attempt (or more than one proof attempt), but uses only one approach</td>
<td>Acknowledges and/or uses more than one proving approach, but only draws on one proof technique</td>
<td>Uses more than one proof technique</td>
</tr>
<tr>
<td><strong>Posing Questions</strong></td>
<td>Recognizes there should be a question asked, but does not pose a question</td>
<td>Poses questions clarifying a statement of a definition or theorem</td>
<td>Poses questions about reasoning within a proof</td>
</tr>
<tr>
<td><strong>Evaluation of Proof Attempt</strong></td>
<td>Examines surface-level features of a proof attempt</td>
<td>Examines an entire proof attempt for logical or structural flow</td>
<td>Examines and revises an entire proof attempt for logical or structural flow</td>
</tr>
</tbody>
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1. Algorithmic
2. Conventional
3. Unconventional
4. Model-based
5. Partly
6. Surface-level
What is creativity?

• Over 100 definitions (Mann, 2006)
• A process of offering new solutions or ideas that are unexpected for the student, with respect to his/her mathematics background or the problems s/he has seen before (Sriraman & Liljedahl, 2006)
How do we “teach” creativity?

- Mathematical creativity in undergraduate teaching
- From Zazkis and Holton (2009):
  - Learner-generated examples (Watson & Mason, 2005)
  - Counter-examples (Koichu, 2008)
  - Multiple solutions (Leikin 2007, 2009)
  - Changing parameters (Brown & Walter, 1983)
- “At the collegiate level, however, very little empirical research has yet described and analyzed the practices of teachers of mathematics” (Speer, Smith & Horvath, 2010, p. 99)
• Levenson (2011, 2013) – Collaborative creativity
  • choosing appropriate tasks,
  • fostering a safe environment where students can challenge norms without fear of repercussion,
  • playing the role of expert participant by providing a breakdown of the mathematics behind a process, and
  • setting the pace, allowing for incubation periods. (Levenson, 2013, p. 273)
Teaching Practices – K-12

- Sriraman (2005) – 5 principles for maximizing creativity

  - The Gestalt Principle: Freedom of time and movement
  - The Uncertainty Principle: Open-ended and/or ill posed problems, tolerating ambiguity
  - The Scholarly Principle: View creativity as contributing to, challenging known paradigms and extending the existing body of knowledge
  - The Aesthetic Principle: Appreciating the beauty of unusual solutions/connections to the Arts and Sciences
  - The Free Market Principle: Encouraging risk taking and atypical thinking
Five Principles

• Gestalt
  • Opportunities to engage in the four-stage creative process (Wallas, 1926; Hadamard, 1945)
    • Preparation, incubation, insight, verification

• Aesthetic
  • Teacher valuing solutions that utilize unusual proving techniques, come from diverse topics of mathematics, or make efficient or elegant solutions
Five Principles (cont.)

• Free Market
  • Creating a classroom environment that allows students to freely input ideas, thoughts, and solutions

• Scholarly
  • Creating a classroom environment “in which students are encouraged to debate and question the validity of… approaches to problems…, be encouraged to generalize the problem and/or the solution, as well as pose a class of analogous problems” (p. 28)

• Uncertainty
  • “Students be exposed to the uncertainty and the difficulty of creating mathematics” (p. 28)
Research Questions

- What teacher actions or practices in the proof-based tertiary classroom might foster students’ perceptions of mathematical creativity?
- Do the five principles apply to undergraduate teaching?
- In particular, how does the CPR on Proving enhance the potential for mathematical creativity in a proof-based classroom according to the five principles?
Data Collection

• Three collections of data
  • Teachers’ impressions (diaries, goals, interviews)
  • Teachers’ instruction (Livescribe™ data, notes)
  • Students’ impressions (online survey, interviews, homework)
• Preliminarily coding teacher actions using the five principles, then using student data to corroborate effectiveness of teacher action
Houssein’s Class - Setting

- 6 students, Number Thy, Fall 2015
- Houssein explained the rubric in one class period (third week)
- Led discussion by asking about subcategories in CPR
- Had co-constructions of proofs in class, and would refer to the rubric to push proving further
- Students were formally asked to use CPR five times in the course
  - Homework as an evaluation tool
  - During the final for extra credit
- 18 students, Combinatorics course, Spring 2016
- Active Learning Hybrid
- Used rubric for “portfolio problems”
  - “much more involved, and the intention is to allow freedom to roam with it in any direction you wish.”
  - One portfolio problem worth three exercises
  - Minimum 3-page essay summarizing the proving process the students used
- Open problems in Combinatorics
14 Students, Transition to proof course, Spring 2016
Utilized rubric language before introducing CPR
Reflections throughout the course asking about mathematical creativity
Showed an attempt from a previous course and attempts from the current students, and had students in group discussion evaluate the students’ work using CPR
“Mathematics at the professional level is full of uncertainty and ambiguity…Creating, as opposed to learning, requires that students be exposed to the uncertainty and the difficulty of creating mathematics” (Sriraman, 2005, p. 28)
Uncertainty Principle

- Mohamed – Portfolio problems were never graded on completion, but rather progress, and some were open problems
- Gail – Not answering correctness or validity questions right away
- Houssein – Getting stuck in class while proving a theorem on two occasions (having to switch techniques) and discussing this episode with students after completing the proof on the third occasion
Uncertainty – Students

• Mohamed – “You could approach different parts of the problem that it wasn’t about getting an answer as much as just kind of exploring relationships…this idea that you could actually kind of like create or notice relationships that are like, like in kind of like new ways rather than just kind of like following a template…

• Gail – “There were times like ‘Well I did this proof, but I’m not sure it’s right because of this’ and she would respond with ‘Well, what do you think class?’ And the class would participate in it…And it’s just, using each other and building off of each other in the class to build what we need, create, made us creative. It built that creative environment for us.
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Instructors valuing solutions that utilize unusual proving techniques, come from diverse topics of mathematics, or make efficient or elegant solutions

- Gail - “...That's the exam 2 ‘solutions’ and I say solutions in quotes because they're not all 100% correct, okay, but it doesn't matter. You know there are still really good ideas in there and that's what I want you to see.”
- Student – “I think tools and tricks especially, because …there’s always different ways to go about it… I feel like with the tools and tricks it’s hard for any two proofs to be exactly alike.”
“…teachers should encourage students to take risks” (Sriraman, 2005, p. 28)

- Houssein - Asking questions using the rubric’s language to encourage students in class
  - Have you made a connection to Theorem T? Or Def A? Have you generated an example to help understand the statement?
- Student - (On being creative in the course)
  “I would say yes… we were given our books to look back…I found one of the questions on the test to be almost exactly like one of the theorems, so I related it directly to that theorem, and it was like two chapters previous, like no one was thinking that far back.”
(a) (5 pts) Create a theorem where the result would contribute to the mathematical knowledge of our classroom community. OR create a theorem about something that you wondered about while working with sets. **You must use at least two sets.** Other symbols that you might use are “=”, “⊆”, “∩”, “∪”, and “∅”. Feel free to use power sets as well (+2 extra credit points if you do!).

(b) (5 pts) Prove your theorem.
Quiz 5: Conjectures created by class

April 2016

Are the following conjectures true or false? If true, prove. If false, show a counterexample or discuss why it’s false. If truth value cannot be determined because there is missing information, then you may add that missing information. For example, you may additional assumptions. You may also change symbols. For example if it’s not true for the two sets to be equal, but it is if one is a subset of each other, then you can change “=” to “⊆”.

1. **Conjecture 1.** Let $S$ and $T$ be sets. Then $P(S) \setminus P(T) \subseteq P(S \setminus T)$.

2. **Conjecture 2.** Let $A$ and $B$ be sets. The $P((A \cap B)^c) = P(A^c \cup B^c)$.

3. **Conjecture 3.** Let $A$ and $B$ be sets. If $A \subseteq B$ and $B \subseteq C$ then $P(C^c) \subseteq P(A^c)$.

4. **Conjecture 4.** Let $A$ and $B$ be sets. If $A \subseteq B$, then $A \subseteq P(B)$.

5. **Conjecture 5.** Let $S$ and $T$ be sets. If $S \subseteq T$, then $T^c \subseteq S^c$. 
• Student of Gail - “She wanted to make sure that the only time when she interfered with the process is if really like she didn’t really see, like nothing else was going on. But that rarely happened. Usually somebody stepped up. Somebody said something that helped to realize that, ‘Wait. What if we did it this way?’ And it changed the entire, it just changes everything. It changes the entire proof altogether.”
• Student of Gail – “Miss Tang asked us to tell us something about ourselves, something that we’re good at. And one of the guys in class stated that he was lazy, I mean efficient. And so efficient kind of became the way of saying that we did it with less work. We were able to work smarter not harder type of thing.”
Pre-Conclusion

- There are signs that using the five principles may shed light on the commonalities of teaching actions between the three case studies.
- There is a spectrum of usage for the principles in the classroom.
- The CPR on Proving is a “vocabulary package” that might be a tool to utilize the five principles:
  - “Tools and tricks”
Future Research

• The CPR allowed students to take a metacognitive look at their own proving process
• “Creative actions might benefit from meta-cognitive skills and vice versa, regarding the knowledge of one’s own cognition and the regulation of the creative process” (Katz and Stupel, 2015, p. 69)
• In-depth investigation of one class (Gail)
“So I think that like everyone’s capable of mathematical creativity. I think that mathematical creativity is not really kind of taught or not made accessible to people, so I think people a lot of times don’t realize that they’re capable of being creative.”

Thank you!

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Questions to Audience…

- How can you “evaluate” your teaching of soft skills like creativity or metacognition?
- Are there either frameworks or literature that may assist the five principles by Sriraman (2005)?
- Is there a quality component to implementing the five principles? Can one “use” a principle without fostering any creativity from the students?